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AN ANALYTICAL STUDY OF AIRCRAFT LATERAL-DIRECTIONAL HANDLING QUALITIES USING PILOT MODELS

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16. Abstract

This study demonstrates a procedure for predicting lateral-directional pilot ratings on the basis of the characteristics of the pilot model and the closed-loop system characteristics. A correlation is shown to exist between experimentally obtained pilot ratings and the computed pilot ratings.

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SUMMARY

An analytical study of the combined pilot-aircraft system has been made to develop a procedure for computing pilot ratings for the lateral-directional response. The results demonstrate the correlation that exists between the computed pilot ratings and the pilot ratings obtained in flight and simulator studies for 66 different aircraft configurations.

In the analytical procedure the pilot is represented by a linear, second-order pilot model; the aircraft by linear equations of motion. Three levels of pilot response are provided by the pilot model. The first level contains a static gain and a second-order lag function with a time constant of 0.2 second. The second level adds a lead-time constant which does not exceed 1 second. The third level presumes a reduction in lag-time constant to as little as 0.05 second. The second and third levels correspond to progressively greater effort by the pilot. Also, the study shows that a suitable pilot-aircraft system response can be defined as follows: for bank-angle control, a stable response with characteristic frequencies greater than 1.9 radians per second; for heading-angle control, a stable response with a characteristic frequency greater than 1.7 radians per second. If these system-response characteristics can be achieved with the first-level pilot model, the aircraft is given a satisfactory pilot rating. If the aircraft characteristics are such that the second-level pilot model is required to achieve the prescribed system response, the aircraft is rated tolerable. If the third-level pilot model is required, the aircraft is rated unacceptable.

INTRODUCTION

Over the last 20 years, papers like references 1 to 4 have suggested (and to a certain extent applied) the idea that a study of a combined mathematical pilot-model aircraft system response be correlated with pilot opinion. Such a study could lead to a better understanding of pilot ratings and aircraft response requirements than could a study of aircraft response alone correlated with pilot opinion. In general, these papers have concluded that the aircraft is satisfactory if the pilot can control the aircraft by operating as a simple amplifier; that if the

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pilot must supply lead to stabilize the system, the aircraft is given a degraded rating. Over the past several years, improvements in the measurements of pilot response when controlling plants or aircraft in a closed-loop manner have led to further application of improved models in the computation and prediction of pilot ratings. References 5 and 6 examine the aircraft longitudinal-control problem, and references 7 and 8 examine the helicopter hover problem. Although these studies vary in their details, they are similar in that pilot ratings were determined by weighing (1) the control technique as expressed by the compensation (lead and lag) that the pilot had to apply to stabilize the vehicle and (2) the response that was obtained in terms of the pilot-vehicle system response. In references 5 and 6 the system response was expressed in terms of system characteristics, and in references 7 and 8 it was expressed as the root mean square error that resulted from a given random signal input. Each of these studies demonstrated the correlation that existed between the pilot ratings obtained from flight and simulator experiments and the pilot ratings obtained by certain analytical techniques. These studies concluded that the analytical method could be used for preliminary design work.

The present study applies these same procedures to the case of the lateral-directional control of aircraft. The same format used in reference 5, which included a linear pilot model and expressed the pilot-aircraft system response in terms of the system characteristics, was used. Sixty-six different aircraft configurations for which pilot ratings had previously been obtained were examined to determine the correlation that existed for the analytical and experimental pilot ratings with a view toward obtaining a simpler set of rules for establishing handling qualities for a given aircraft.

The effects of control saturation, extremely high or low control sensitivity, and the use of rudder control are not considered in this study. Higher order systems which might result from power control systems and shaped augmentation systems are not considered either, but it is clear from this study and from the results in reference 5 that the method given here could be used to compute pilot ratings for such systems.

SYMBOLS

c	command
e	displayed error, rad
g	gravity, 9.81 m/sec^2
I_X, I_Z	moment of inertia, $kg\text{-}m^2$
I_{XZ}	product of inertia, $kg\text{-}m^2$

K_1	pilot-model static gain	
K_ϕ, K_ψ	inner loop and outer loop pilot-model static gain	
L	rolling moment, N-m	
L_i'	$= \left(1 - \frac{I_{XZ}^2}{I_X I_Z}\right)^{-1} \left(L_i + \frac{I_{XZ}}{I_X} N_i\right)$	$(i = \beta, p, r, \delta_a, \delta_r)$
L_p	$= \frac{1}{I_X} \frac{\partial L}{\partial p}$ per sec–rad	
L_r	$= \frac{1}{I_X} \frac{\partial L}{\partial r}$ per sec–rad	
L_β	$= \frac{1}{I_X} \frac{\partial L}{\partial \beta}$ per sec^2–rad	
L_{δ_a}	$= \frac{1}{I_X} \frac{\partial L}{\partial \delta_a}$ per sec^2–rad	
m	mass, kg	
N	yawing moment, N-m	
N_i'	$= \left(1 - \frac{I_{XZ}^2}{I_X I_Z}\right)^{-1} \left(N_i + \frac{I_{XZ}}{I_Z} L_i\right)$	$(i = \beta, p, r, \delta_r, \delta_a)$
N_p	$= \frac{1}{I_Z} \frac{\partial N}{\partial p}$ per sec–rad	
N_r	$= \frac{1}{I_Z} \frac{\partial N}{\partial r}$ per sec–rad	
N_β	$= \frac{1}{I_Z} \frac{\partial N}{\partial \beta}$ per sec^2–rad	

N_{δ_a} $= \frac{1}{I_Z}\frac{\partial N}{\partial \delta_a}$ per sec^2-rad

N_{δ_r} $= \frac{1}{I_Z}\frac{\partial N}{\partial \delta_r}$ per sec^2-rad

p,r,q roll, yaw, and pitch rates, rad/sec

q pitch rate, rad/sec

s Laplace operator, per sec

T_R aircraft roll-time constant, sec

T_S aircraft spiral mode-time constant, sec

T_1 pilot-model lag-time constant, sec

T_2 pilot-model lead-time constant, sec

V resultant inertial velocity, m/sec

Y side force, N

Y_p $= \frac{1}{mV}\frac{\partial Y}{\partial p}$ per rad

Y_r $= \frac{1}{mV}\frac{\partial Y}{\partial r}$ per rad

Y_β $= \frac{1}{mV}\frac{\partial Y}{\partial \beta}$ per sec-rad

α_o angle of attack, rad

β sideslip angle, rad

δ control deflection, rad

δ_a,δ_r aileron and rudder control deflection, rad

ζ_d aircraft Dutch roll damping ratio

ζ_{RS} aircraft roll-spiral mode damping ratio

$\zeta_{1,2,3}$ system-response mode damping ratio

ζ_ϕ damping ratio appearing in the numerator quadratic of ϕ/δ_a transfer function

θ,ϕ,ψ pitch, roll, and heading angle, rad

ω_d aircraft Dutch roll frequency, rad/sec

ω_{RS} aircraft roll-spiral mode frequency, rad/sec

$\omega_{1,2,3}$ system-response mode frequencies, rad/sec

ω_ϕ undamped natural frequency appearing in the numerator quadratic of ϕ/δ_a transfer function, rad/sec

Dots over symbols indicate derivatives with respect to time.

SYSTEM DESCRIPTION

Pilot Models

The procedure used in this investigation was to analyze both roll and heading system response. By studying both of those responses, this paper considers nearly all lateral-directional functions which a pilot attempts with an airplane. Both time-history responses to step commands and system characteristics obtained by factoring the system characteristic equation were obtained. The system was represented by the three degree-of-freedom lateral equations of motion for the aircraft and a pilot model. For single-loop control tasks (e.g., control of bank angle), the pilot model used in this investigation consisted of the following elements: a static gain and a lead-time constant, which together constitute a mentally commanded control deflection; and second-order, critically damped lag function, which represents the dynamic response of the arm in executing this command. In transfer function form the model is:

$$\frac{\delta}{e} = \frac{K_1(1 + T_2 s)}{(1 + T_1 s)^2}$$

Reference 9 shows that there is a certain order to adjustment and certain limitations in the adjustments that a pilot makes in his control response. As the difficulty of control is increased by increasing the lag in the vehicle or plant, pilots increase their lead from a time constant T_2 of zero to 1 second. With further increase in plant lag the pilots decrease their second-order lag-time constant T_1 from 0.2 second to as low as 0.05 second. These variations in pilot response are used as part of the scheme to determine pilot ratings of aircraft.

For multiloop control tasks (e.g., the control of heading), previous experiments like those described in reference 10 have shown that an appropriate model for representing the pilot consists of two transfer functions in series, with one transfer function in each control loop (illustrated in fig. 1). With this arrangement, an error in heading generates a bank-angle command. The inner loop generates a control deflection which brings about the commanded bank angle. The inner-loop pilot transfer function is the same as that described in the previous paragraph for the single-loop bank-angle control. The outer-loop pilot transfer function consists of a static gain and a lead-time constant. However, since outer-loop pilot model lead was not used in this report, it is not shown in figure 1.

Criteria for Estimating Pilot Ratings

The results from references 5, 9, and 10 which were discussed in the previous paragraphs have led to a proposed scheme of pilot-response levels and system-response characteristics which, it is hypothesized, should correspond to pilot ratings. These rules have already been studied for the longitudinal aircraft control problem (ref. 5), and are studied for the aircraft lateral-control problem in this investigation. The intention of this study is only to classify the different configurations according to the pilot rating classification of satisfactory (ratings from 1 to 3.5), tolerable (ratings from 3.5 to 6.5), unacceptable (ratings from 6.5 to 9), and uncontrollable (a rating of 10). A more complete description of the rating scale is given in reference 11, and the summary chart for the scale is presented in figure 2.

The logic for pilot rating determination for single-loop control tasks such as bank-angle control, based on the concepts outlined here, is as follows:

(1) The pilot prefers to operate as a simple amplifier with a lag-time constant of 0.2 second and zero lead. This is a very simple response achieved in a relatively leisurely manner. It is hypothesized that if what are subsequently defined as "suitable" system-response characteristics can be achieved with this pilot response, the configuration is judged satisfactory (pilot rating of 1 to 3.5).

(2) If moderate compensation is required on the part of the pilot to achieve a suitable system response, the compensation takes the form of a lead with a time constant of as much as 1 second, with the lag-time constant maintained at 0.2 second. In order to supply this lead, the pilot performs the additional mental task of differentiating the displayed information.

If a suitable system response can be achieved with this pilot action, the configuration is rated tolerable (a rating of 3.5 to 6.5).

(3) If further compensation is required to achieve a suitable system response, it is supplied by reducing the lag-time constant, with values as low as 0.05 second being obtainable. To achieve this response, the pilot must perform not only the additional mental task of differentiating the displayed information, but he must also increase his muscle tension to obtain the reduced lag in control deflection. If this compensation is required to achieve a suitable system response, the configuration is rated unacceptable (a rating of 6.5 to 9).

(4) If a suitable system response is not obtained with (3), the configuration is rated uncontrollable (a rating of 10).

For outer-loop control (e.g., the control of heading), two levels of preferred response for the outer-loop pilot model, to be used in conjunction with the inner-loop pilot model, are:

(1) The outer-loop pilot model should preferably be a simple amplifier with zero lag. If this type of outer-loop pilot-model response is all that is required to obtain a suitable system response, then the pilot rating is determined by the lead and lag required in the inner-loop pilot model to provide a suitable heading response.

(2) If compensation is required in the outer-loop pilot model, it takes the form of lead. Reference 10 has shown that the outer-loop pilot lead-time constant can be as high as 10 seconds. If lead is required to obtain a suitable system response, some (as yet undefined) reduction in pilot rating is required. The addition of lead in the outer-loop pilot model was not required in this investigation.

"Suitable" System-Response Characteristics

In addition to the various levels of pilot response, it is necessary to define a suitable system response in order to have a formula that can be used to compute pilot ratings. This definition defines the response that the pilot expects the total system to exhibit. It is proposed that the form of this definition can be a specification of a system time characteristic. This time characteristic expresses either (1) the time required for a knowledgeable pilot to perceive the steady state condition that is eventually achieved following a step command, or (2) the exactness with which a sinusoidal command is followed. For a system in which the dominant characteristic is oscillatory, the time characteristic is the time required for one cycle; and, of course, the response must be stable. If the dominant characteristic of the system is a first-order term, the time characteristic is the time constant of the first-order term, and all of the higher frequency oscillatory modes must be stable.

In the study of longitudinal control (ref. 5), it was found that a time characteristic of 2.5 seconds for pitch-angle control and 5 seconds for altitude control were the values to use. In reference 9, which reports on the measurement of pilot response in single-axis control tasks for a number of subjects and for a very wide range of vehicle dynamics, it was found that the pilots controlled the system so that the time characteristic was between 3 and 1.5 seconds (frequencies of 2 to 4 radians per second and real roots of 0.4 to 0.6 radian per second).

The present investigation defines a system time characteristic that applies to lateral-directional aircraft control tasks. Separate and different values are expected to be necessary for single-loop roll-control tasks and multiloop heading-control tasks. Also, the values are expected to be different from those that applied to the longitudinal control. The values required for other types of vehicles, helicopters, for example, might also be different. However, the values determined in this study should apply to most aircraft roll- and heading-control tasks.

Pilot Rating Data

The experimentally obtained pilot-rating data that were used to test the correlation that could be obtained with the pilot models were obtained from references 12, 13, and 14. The configurations covered in reference 12 included systematic variations in roll-time constant T_R and the ratio of roll to sideslip ϕ/β with variation in the yaw due to aileron parameter $N'_{\delta_a}/L'_{\delta_a}$ for each combination of T_R and ϕ/β. Reference 13 covered systematic variations in Dutch roll frequency ω_d, and Dutch roll damping ratio ζ_d with variations in the yaw due to aileron parameter $N'_{\delta_a}/L'_{\delta_a}$ for each combination of ω_d and ζ_d. Reference 14, which was a study conducted with a fixed-base simulator, covered a wide range of the frequency of the coupled roll-spiral aircraft mode of motion. Taking selected examples from each of these references makes possible an examination of a wide variety of lateral aircraft response characteristics. A listing of the stability derivatives for each configuration examined in this study is given in table I. This list uses the same designation as the reference reports and the case number employed in this report. The roll-control effectiveness L'_{δ_a} was arbitrarily selected to be 1.0 in all cases. The pilot-rating data from these various sources are given in column 3 of table II. Ratings of references 12 and 13 are from a single pilot, and ratings of reference 14 are from two pilots.

Equations of Motion

The stability derivatives from the various references were used with the following body axis equations of motion:

$$-\dot{\beta} + Y_\beta \beta - r + Y_r r + \alpha_o p + Y_p p + \frac{g}{V}\phi = 0$$

$$N'_\beta \beta - \dot{r} + N'_r r + N'_p p + N'_{\delta_r}\delta_r + N'_{\delta_a}\delta_a = 0$$

$$L'_\beta \beta + L'_r r - \dot{p} + L'_p p + L'_{\delta_a}\delta_a = 0$$

The bank angle ϕ and the heading ψ were obtained from the expressions

$$\dot{\phi} = p$$

$$\dot{\psi} = \frac{g}{V}\phi$$

The $\dot{\phi}$ equation was obtained from a linearization of one of the Euler equations. The $\dot{\psi}$ equation was obtained from the following simplifications for the relations for a coordinated turn for which $\dot{\theta} = 0$. The r Euler equation is $r = -\dot{\theta} \sin \phi + \dot{\psi} \cos \theta \cos \phi$; in a steady horizontal turn this equation becomes $r = \dot{\psi} \cos \phi \cos \theta$. For small θ this expression is $r \approx \dot{\psi} \cos \phi$. For dynamic equilibrium in a turn which is coordinated so that $\beta = 0$

$$mg \sin \phi = mrV$$

Substituting for r gives

$$mg \sin \phi \approx m\dot{\psi} \cos \phi V$$

or

$$\dot{\psi} \approx \frac{g}{V} \tan \phi$$

This expression linearizes to $\dot{\psi} \approx \frac{g}{V}\phi$. This expression is approximately correct for coordinated turning maneuvers at small ϕ and θ. Another expression which could be used for heading rate is $\dot{\psi} = r$. This expression may be obtained by linearizing the Euler equations for a constant θ without using the coordinated turn expression for dynamic equilibrium. The steady state terms of the heading to aileron deflection transfer functions using each of these expressions are similar. The higher order terms are different. As a result, the computed changes in heading to a step aileron deflection are similar except for some transient effects.

There was a noticeable difference in the system response when the pilot model was included in the system for these two expressions for heading. The system frequencies were different by 25 percent more or less, and the pilot gains for neutral dynamic stability were different. The pilot gains do not enter the handling qualities criteria. In about 13 check cases done, using both expressions, similar pilot ratings were obtained. The results of this paper are based on the relation $\dot{\psi} = \frac{g}{V}\phi$, but the use of either of the heading relations gives results that in some cases validate the pilot-rating concept investigated here. The particular system-response frequencies used in the handling quality rules would change, however, depending on which expression for heading is used.

To determine the pilot-model aircraft system characteristics, the following pilot equations are added to the aircraft equations given above.

For roll control the equation is

$$s^2\delta_a = -\frac{K_\phi}{T_1^2}(\phi - \phi_c) - \frac{K_\phi T_2 s\phi}{T_1^2} - \frac{2s\delta_a}{T_1} - \frac{1\delta_a}{T_1^2}$$

For heading control, add

$$\phi_c = -\frac{g}{V}K_\psi(\psi - \psi_c)$$

The combined pilot aircraft equations result in transfer functions of the form

$$\frac{\phi}{\phi_c} = \frac{K_\phi(1 + T_2 s)L'_{\delta_a}\left(s^2 + 2\zeta_\phi\delta_\phi s + \omega_\phi^2\right)}{\left(s^2 + 2\zeta_1\omega_1 s + \omega_1^2\right)\left(s^2 + 2\zeta_2\omega_2 s + \omega_2^2\right)\left(s^2 + 2\zeta_3\omega_3 s + \omega_3^2\right)}$$

The closed-loop system data given in this paper correspond to this form.

ANALYSES AND RESULTS

Determination of Dominant Mode Frequency

According to the hypothesis of this paper, a given aircraft in a given flight condition is rated satisfactory if suitable closed-loop system characteristics can be achieved with a pilot model with no lead and a 0.2 second lag-time constant; the aircraft is rated tolerable if these same system characteristics can be achieved with a 1 second lead-time constant. No restriction is placed on the magnitude of the pilot-model static gain used. In addition, suitable system response is at least neutrally stable with a time characteristic of around 2 seconds. To determine the validity of this hypothesis for the lateral response of aircraft, and at the same time to determine the system time characteristic that provides the best correlation with the experimental data, an analog study was performed. For each configuration, the system response to a step bank-angle change command, using aileron control only, was computed for a series of increasing values of pilot-model static gain, both with and without lead, to determine the highest static gain for which a neutrally stable response could be obtained. An analytical determination was then made of the system characteristics for these highest gains. After the roll responses were obtained, a similar procedure was followed to obtain the heading response for some of the configurations.

As an illustration of the results of the analog computation, case 19 is shown in figure 3. Note that there are two dominant oscillatory modes of motion in this system response. As the pilot-model static gain is increased, the period of these oscillatory modes of motion decreases, and the modes become less stable. When lead is added to the pilot model, the damping of the system response is increased, and it is possible to increase the static gain further and still maintain a stable response. This change in system response is the normal type of variation with pilot-model gains. Most of the configurations exhibited this type of variation, but in some cases, which are mentioned later, a different type of variation occurred.

Since the lowest frequency mode is often difficult to distinguish in these time histories, the system characteristic factors of the pilot model and aircraft system were also obtained to acquire a better evaluation of this lowest frequency. This dominant mode frequency for a neutrally stable response for roll control, both with and without pilot lead, is plotted for appropriate cases against the experimentally determined pilot ratings in figure 4. Some configurations for which heading control and pole-zero cancellation were determining factors are not plotted in figure 4, but are discussed later. The figure shows that the best correlation between computed pilot rating and the experimental pilot rating is obtained if the criterion for system frequency is set at 1.9 radians per second. This criterion is the same as requiring that the system time characteristic be less than 3.3 seconds. The aircraft rated from 1 to 3.5 have frequencies higher than 1.9 radians per second with no pilot lead. The aircraft rated between 3.5 and 6.5 have frequencies lower than 1.9 radians per second with no pilot lead and higher than 1.9 radians per second with pilot lead. This value was, therefore, selected for use in subsequent determination and discussion of pilot rating for roll control. It should be noted that this system frequency criterion value of 1.9 radians per second is different from that used in the study of aircraft longitudinal control, where a value of 2.5 radians per second was used.

Similar results from an analysis of heading control for selected examples that cover the complete range of pilot ratings are presented in figure 5. The multiloop pilot model shown in figure 1 was used in computing heading response. The equation used for heading was the coordinated turn relation $\dot{\psi} = \frac{g}{V}\phi$. From figure 5 it can be clearly seen that a system frequency requirement of 1.7 radians per second gives the best correlation between computed and experimentally determined pilot ratings. Therefore, this value of 1.7 radians per second is used in all further computations of heading control. If the formula $\dot{\psi} = r$ had been used for heading instead of $\dot{\psi} = \frac{g}{V}\phi$, the value for the system dominant mode frequency would have been 1.5 radians per second.

The frequencies plotted in figure 5 are supposed to represent the best system response that can be obtained for the particular aircraft pilot-model combination. However, since the adjustment of two pilot-model static gains is involved in the heading response, the process of obtaining the best possible response is more complicated than securing the best response for roll control. It was sufficient for this analysis to determine these gains within a factor of two. The selection of the best response was based on a subjective judgment of the computed time histories and the system characteristic factors. However, the best response was fairly well defined, and it is felt that the responses presented are at least very close to an optimum response.

Consideration of Rudder Control

For lateral control, compensation can be supplied by means other than adding lead to the aileron control. For example, if the configuration had either proverse or adverse yaw due to aileron, either of which would cause a reduced estimate of pilot rating, the rudder could be used, and very often is used, to cancel the yaw due to aileron by coordinating the rudder control with the aileron. In other cases it is likely that use of the rudder to regulate yawing velocity or sideslip would improve the system response. However, these alternate methods for obtaining the required compensation would also require additional effort on the part of the pilot, and if used, such methods would result in a pilot rating that would reflect this requirement for additional effort. The rudder is used for stability compensation primarily when the configuration is so difficult to control that it is rated between 7 to 10. Because the emphasis in this study is to establish the 3.5 and 6.5 pilot rating boundaries, and because it is felt that rudder control is not essentially involved in these boundaries, no further consideration is given to rudder control.

Consideration of Small Real Roots

In the study of longitudinal control presented in reference 5, there were some cases in which a first-order system characteristic, or real root, was a critical factor because of its slow response. In this study there were no examples in which the real roots were a critical factor. However, in any case where a real root less than about 0.3 radian per second appears, this consideration may be important.

Computed Pilot Ratings

Roll control.- A detailed discussion of all the examples considered in this study follows. For convenience, the pilot ratings are computed as though these configurations were new designs. Thus, the factors to be considered are illustrated.

A complete listing of the results of the bank-angle control analysis is presented in table II. For convenience, the cases are listed according to the rank of the experimentally determined pilot rating. The table presents the closed-loop system-response characteristics for the highest pilot-model static gain for which a neutral, or nearly neutral, stable response was obtained. If a good response was obtained without pilot lead, then that response is presented.

In the remaining cases, the response with pilot lead is presented. The vehicle open-loop response characteristics are also presented.

The process of computing pilot ratings for roll control is illustrated with selected examples in the following table:

Case	Configuration		Pilot model gains			Closed-loop characteristics							Estimated pilot rating	Pilot rating from refs. 12, 13, 14
						Oscillatory						Real roots		
	Group	$N'_{\delta_a}/L'_{\delta_a}$	K_ϕ	T_2	T_1	ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3			
1	BB-2	0	11	0	0.2	2.0	0.09	2.8	0.001	6.9	0.92		1 to 3.5	2
19	AB-3	0	8	0	0.2	1.8	0.25	2.7	-0.02	6.5	0.94			
			12	1	.2	2.3	.22	5.5	.04			-0.9, -10.6	3.5 to 6.5	5.5
29	BB-2	-0.07	8	0	0.2	1.2	0.35	3.0	0	6.6	0.93			
			20	1	.2	1.6	.20	6.8	.02			-0.9, -12		
			4	1	.05	2.0	.26	9.0	.76			-0.4, -27.0	6.5 to 9	6.5
33	BB-1	-0.10	3	0	0.2	0.78	0.64	3.0	0.01	6.3	0.94			
			16	1	.2	.98	.32	3.7	-.04	6.6	.96			
			4	1	.05	1.9	.41	8.8	.76			-0.27, -27.0	6.5 to 9	7.0

The table shows that for case 1, the required system response is obtained with a pilot lead of zero; therefore, the computed pilot rating should be between 1 and 3.5. This result agrees with the experimentally obtained pilot rating. For case 19, the required system response was not obtained with a lead of zero, but with pilot lead; therefore, this case should have a rating between 3.5 and 6.5. Again, this result correlated with the experimental result. For cases 29 and 33 the required system response is obtained only with pilot lead and with the lag-time constant reduced to 0.05 second, indicating that the pilot rating should be between 6.5 and 9; the result again correlates with the experimental data. Note that further consideration for the case 29 configuration occurs later. This last step in the analysis, the reduction of the lag-time constant to 0.05 second, was not carried out for the remainder of the configurations. Instead, if the required system response was not obtained by adding the 1-second lead-time constant, it was assumed without further analysis that the pilot rating should be between 6.5 and 9.

Among the well-documented effects of aircraft response factors on pilot ratings for which the pilot ratings (computed by the method of this report) agree with the experimental pilot ratings are the effects of roll-time constant T_R, roll-sideslip ratio ϕ/β, adverse yaw, and roll-spiral coupling. (Cases with proverse yaw are discussed later.) This correlation is illustrated with the selected examples listed in the following table. Examples like these, where the correct answer is provided by the simple rule that the lowest system characteristic be greater than 1.9 radians per second, accounted for 13 of the 32 cases examined which had

Effect of roll-time constant

Case	Configuration		T_R, sec	T_2, sec	Closed-loop characteristics							Computed pilot rating	Experimental pilot rating
	Group	$N'_{\delta_a}/L'_{\delta_a}$			ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots		
5	BA-2	0	0.15	0	1.95	0.13	3.5	0	9.0	0.92		1 to 3.5	3.5
1	BB-2	0	.4	0	2.0	.09	2.8	0	6.9	.92		1 to 3.5	2.0
15	BC-2	.02	1.4	1	2.4	.12	4.5	.01			-1, -9	3.5 to 6.5	5.0

Effect of ϕ/β

Case	Configuration		ϕ/β	T_2, sec	Closed-loop characteristics							Computed pilot rating	Experimental pilot rating
	Group	$N'_{\delta_a}/L'_{\delta_a}$			ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots		
4	AB-2	0	1.5	0	2.0	0.09	2.6	0.05	6.6	0.93		1 to 3.5	3.0
1	BB-2	0	5	0	2.0	.09	2.8	0	6.9	.92		1 to 3.5	2.0
18	CB-2	0	13	1	2.0	.20	4.9	.12			-0.8, -9.0	3.5 to 6.5	5.5

Effect of adverse yaw

Case	Configuration		T_2, sec	Closed-loop characteristics							Computed pilot rating	Experimental pilot rating
	Group	$N'_{\delta_a}/L'_{\delta_a}$		ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots		
1	BB-2	0	0	2.0	0.09	2.8	0	6.9	0.92		1 to 3.5	2.0
29	BB-2	-.07	1	1.6	.20	6.4	.02			-0.8, -11	6.5 to 9	6.5
15	BC-2	0.017	1	2.4	0.12	4.5	1.01			-1, -9	3.5 to 6.5	5
30	BC-2	-.066	1	1.4	.19	4.9	.02			-1, -9	6.5 to 9	6.5
4	AB-2	0	0	2.0	0.09	2.6	0.05	6.6	0.93		1 to 3.5	3
45	AB-2	-.16	1	2.0	.16	6.3	.01			-0.9, -11	3.5 to 6.5	7

Effect of roll-spiral coupling

Case	Configuration		T_2, sec	Closed-loop characteristics							Computed pilot rating	Experimental pilot rating
	Group	$N'_{\delta_a}/L'_{\delta_a}$		ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots		
2	I-1		0	1.9	0.37	2.3	0	6.9	0.93		1 to 3.5	2.0
7	I-17		1	1.9	.68	4.9	0			-1.1, -9.9	3.5 to 6.5	3.5
27	I-11		1	1.9	.64	4.2	.03			-1, -9	3.5 to 6.5	6.5
35	I-21		1	1.4	-.03	2.9	.46			-1, -7	6.5 to 9	7

pilot ratings between 1 and 6.5. An examination of the remaining cases showed that many of them contained pole-zero cancellation in the system response. That is, the lowest frequency in the system characteristic was very nearly the same as the zero of the open-loop aircraft response to an aileron input. In these cases this low frequency characteristic does not appear in the system output to a bank-angle command. This effect is illustrated in figure 6 which presents the response to a step bank-angle command for cases 9 and 20. These two cases have the pole-zero cancellation in the system response; cases 54 and 55 (fig. 6) do not. For example, table II illustrates that, for case 20, the lowest system frequency characteristics are $\omega_1 = 0.95$ and $\zeta_1 = 0.23$. However, those characteristics are very nearly the same as those of the system zero, $\omega_\phi = 0.93$ and $\zeta_\phi = 0.13$, and pole-zero cancellation results. The system mode of motion with the 6.6-second period does not appear in the time history. On the other hand, for case 55 the lowest system frequency characteristics are $\omega_1 = 0.69$ and $\zeta_1 = 0.35$. These characteristics are not close enough to the zero characteristics $\omega_\phi = 0.64$ and $\zeta_\phi = 0.15$ to have pole-zero cancellation. As a result, the mode of motion with the 10-second period is present in the time history.

When pole-zero cancellation occurs, the low frequency system characteristic can be disregarded when determining the pilot rating. In all of these cases the next lowest frequency in the system characteristic was always greater than the required 1.9 radians per second. Therefore, the pilot rating should be 1 to 3.5 if no lead is required to obtain the cancellation and 3.5 to 6.5 if lead is required. An approximate criterion for cancellation is that the sum of the difference of the pole and zero frequency and the difference of the pole and zero damping ratio be less than 0.12. By including this modification to the rules for computing pilot rating, 23 of the 32 cases examined with pilot ratings from 1 to 6.5 can be correctly computed. The modified rule also provides the correct computed pilot rating for 12 of the 33 configurations that were rated between 6.5 and 9 by the pilots.

Of the remaining cases that cannot be explained with the modified rule, a large number were cases with proverse yaw, all of which cases were computed to be better than the ratings given by the pilots. Two of the cases were in the 3.5 and 6.5 pilot rating range, and 11 were in the 6.5 to 9 range. Two cases in which the discrepancy was large were cases 38 and 39, for which the modified rule would compute pilot ratings of 1 to 3.5, but which the pilots rated 7. To find the reason for this discrepancy, these cases were examined further. A careful study was made of the heading response.

Heading control.- A list of the results for the computation of heading response for selected configurations covering the complete range of pilot ratings is given in table III. The multiloop pilot model shown in figure 1 was used in this computation. These results show a pattern very similar to those obtained for the roll response. For those configurations that were rated between 1 and 3.5 by the pilots, a neutrally stable response with the lowest frequency not less than 1.7 radians per second (a time characteristic not greater than 3.7 seconds) was obtained. This value is a very reasonable criterion to use as a suitable response for heading control. For those configurations rated between 3.5 and 6.5 by the pilots, this criterion could be met only by adding lead to the inner-loop pilot model. For case 14, the occurrence of a pole-zero cancellation in the heading response had to be considered. For configurations rated between 6.5 and 9, it was necessary to reduce the pilot-model lag to meet the criterion. Thus, by establishing a criterion for heading response that the lowest system frequency be greater than 1.7 radians per second, the computation of variations in pilot ratings for heading control is possible, just as such a computation was possible for roll control.

The results for configurations with zero or adverse yaw listed in table III are computed pilot ratings which are the same as those computed for roll control. However, when cases with proverse yaw were considered, it was found that the computed pilot rating based on heading control was different than that computed for roll control. There were also unusual variations with pilot gain in the proverse yaw cases.

The results that have unusual variations of system-response frequency with pilot lead and static gains are shown in table IV. Normal cases 19 and 25, in which the addition of lead results in an improvement in system response, are presented together with five cases with proverse yaw. Note that in the latter cases the heading control system response with no pilot lead just barely meets or is less than the established criterion. When lead is added in these cases, the system response is degraded. The effect of reducing the pilot model lag is also shown. For case 38, the heading criterion is met with lead and reduced lag. The other configurations show an improvement in heading response (in some cases only marginal improvement) with the combination of reduced lag and no lead. Therefore, heading response rather than roll response appears to be the critical factor with these configurations. The difficulty seems to be a combination of unusual variation of system frequency with gain and a failure to meet the system-response criterion. For these reasons these configurations are given computed ratings from 6.5 to 9.

There was also some indication that unusual variations with gain occurred in the roll response for configurations with proverse yaw. For case 16, the addition of lead resulted in a very noticeable decrease in the rate of roll response to a roll command. The decrease in rate of roll response was very noticeable in the computed time history and corresponded to a very low real root in the system characteristic. Case 38 (table V) exhibited an unusual variation with static gain. The normal variation was that the system response would become increasingly unstable with an increase in static gain. For case 38, the system was stable for low gain, unstable for an intermediate gain, and stable for a high gain as is shown in the variation of ζ_1.

These unusual response variations with pilot gains in the roll response may have contributed to the poor pilot ratings for configurations with proverse yaw. The most clearly demonstrated difficulty with these configurations, however, was the unusual variation, with pilot lead, in the heading response.

Other cases.- At the extreme end of the rating scale were three cases which the pilots rated 9 to 10, and for which no stable solution for roll control was obtained with the pilot models. (See table II.) The computation, therefore, indicates that these configurations are the worst of the configurations considered, and should be given ratings of 10.

Of the remaining 14 cases of table II, 4 were similar in that the lowest system frequency for roll control was well damped. One of these, case 32, was given a very wide spread in rating, varying from 4.5 to 7, by the pilots. The computed results indicate the system-response characteristics for these configurations are unusual in that the lowest frequency mode was well damped. This factor may contribute to the lack of agreement between the computed results and the experimental results, and to the uncertainty in experimental results for case 32.

In case 61 pole-zero cancellation occurred. Here the computed result did not agree with the experimental result. In reference 13 the pilot subjects explained that this configuration was given a poor pilot rating because of the response in turbulence; in nonturbulent conditions, the rating would have been much better. This configuration appears to be very sensitive to gusts, and there is an unusually large increment in pilot rating because of gust response. The present analysis does not, of course, uncover such an unusual sensitivity to gusts.

In the remaining 9 cases in which the computed pilot ratings did not correlate with the experimental results, there did not appear to be any logical reason for the discrepancy. However, because the discrepancy was not large for any of the configurations, it could not be considered invalidating.

The results of the computation of pilot ratings for all the aircraft configurations studied, some of which are for both roll and heading control, are presented in table VI, together with a comment on the pertinent factors involved in each case. This table provides a convenient summary and reference list for all the cases studied.

Piloted simulation study.- In this study a condition of zero damping for the system response is used as a common point for comparing different configurations. The selection of this point for comparison is not meant to imply that a pilot would control the system with zero damping. To illustrate the system response that pilots do generate, some time histories are presented in figures 7 to 9. These time histories were obtained with a fixed-base simulator. The display on the simulator was a three degree-of-freedom, all-attitude indicator; the controller was a conventional center stick and rudder pedals; and the aircraft was represented by the five degree-of-freedom equations of motion presented in the appendix. Airspeed was held constant. The pilots were asked to perform step changes in bank angle and in heading angle. They did these maneuvers with three different configurations: a satisfactory one, BB-2, $N'_{\delta_a}/L'_{\delta_a} = 0$; an unacceptable configuration with proverse yaw, BB-2, $N'_{\delta_a}/L'_{\delta_a} = 0.07$; and an unacceptable configuration with adverse yaw, BB-1; $N'_{\delta_a}/L'_{\delta_a} = -0.10$. A computed response using the pilot model form considered in this paper is also shown in figures 7 to 9. The gains were selected to approximate the time histories obtained with the pilots in control rather than to obtain the neutrally stable response used in the pilot rating computation. The good match throughout the motion for the 3 cases considered shows the pilot model to be a good representation of the action of the pilot. The degradation from the first case to the last two cases is reproduced, and the detailed features of these responses are generally similar in the computed runs and the piloted runs.

While the agreement between the computed time histories and the piloted runs is good, the small differences are in themselves interesting. In most of the piloted cases, the system bank-angle response shows less damping than the computed time histories shown in figures 7 to 9. This difference indicates that the different pilots varied in their selection of their gains so that the damping was between that presented in the figure and the zero damping used in the handling qualities criterion. The heading response is, in some piloted cases, slower than the computed run. Some of the pilots, particularly subject M, placed a limit on bank angle when controlling heading, and the system response shows the effect of this saturation.

Pilot response was also measured with a random signal disturbance. In this task the pilot controlled bank angle while a white noise rolling-moment signal was added to the roll equation of the aircraft. The measurements were made by using the parameter tracking method described in reference 5, and are shown in table VII. These gains agree with those required to compute the step responses of figures 5 to 7 in that the relative changes with configurations are duplicated. These gains also indicate that approximately the same gains are used by the pilots both for control in the presence of a random signal and for step commands.

CONCLUDING REMARKS

Aircraft lateral handling qualities have always been very difficult to define in a generally useful way. This difficulty can be explained by considering the many forces and moments involved in the lateral motion of aircraft. Some of these forces and moments are of nearly equal magnitude and importance. As a result, small combined changes in certain forces and moments result in substantial changes in response. Attempts to classify the various combinations based on aircraft response have led to a large and complicated set of rules. Despite their complexity, these rules do not completely satisfy the need for handling qualities specification. The attempt to analyze lateral handling qualities given here is based on pilot limitations and their interaction with the aircraft. This concept has the potential of simplifying the classification of lateral handling qualities.

The criteria derived in this investigation to determine lateral handling qualities follow:

1. If what is defined as a suitable system response to bank-angle and heading commands can be obtained for the combination of aircraft and pilot model with no lead included in the model, then the aircraft can be rated satisfactory (a pilot rating between 1 and 3.5); if pilot lead is required to obtain a suitable system response, the aircraft can be rated tolerable (a pilot rating between 3.5 and 6.5); if a suitable system response cannot be achieved with pilot

lead, and further compensation, such as reducing pilot lag, is required, the aircraft is rated unacceptable (a pilot rating between 6.5 and 9). In examining a configuration, both roll and heading control should be considered in determining pilot rating, as it is not always clear which consideration establishes the highest (least satisfactory) rating.

2. A suitable system response is stable, and for roll response has all system characteristic frequencies larger than 1.9 radians per second, and for heading response has all system characteristic frequencies larger than 1.7 radians per second for the heading equation approximation used in this analysis.

3. If pole-zero cancellation occurs in the system bank-angle and heading responses, then the characteristic frequency that is involved in the cancellation can be disregarded in determining the suitability of the system characteristics.

4. If adding lead to the pilot model does not result in an improvement in system response, an unusual difficulty for the pilot occurs, and the pilot rating is noticeably worse.

The examples studied in this report, in general, support these criteria. However, the examples also show that the application of these rules to obtain an absolute prediction of pilot rating may be difficult. Difficulties occur particularly in regard to determining whether pole-zero cancellation is sufficiently complete, and in regard to the increment in pilot rating that will result from unusual variations in system response with pilot gains changes. It is believed, however, that the analyses proposed can provide a good indication of handling qualities and can provide a very useful insight that can be of considerable aid in guiding further simulation and flight experiments, and in understanding the results of these experiments.

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APPENDIX

EQUATIONS OF MOTION FOR THE PILOTED SIMULATION STUDY

The equations of motion used for the piloted simulation experiment were:

$$a_x = 0$$

$$a_y = V_{x_o} Y_\beta \beta$$

$$a_z = -V_{x_o}\left(L_\alpha \alpha + L_o\right)$$

$$\dot{p} = L_p' p + L_\beta' \beta + L_r' r + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e} \delta_e$$

$$\dot{r} = N_r' r + N_\beta' \beta + N_p' p + N_{\delta_r} \delta_r + N_{\delta_a} \delta_a$$

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \frac{r \cos \phi + q \sin \phi}{\cos \theta}$$

$$l_1 = \cos \psi \cos \theta$$

$$m_1 = \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi$$

$$n_1 = \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi$$

$$l_2 = \sin \psi \cos \theta$$

$$m_2 = \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi$$

$$n_2 = \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi$$

$$l_3 = -\sin \theta$$

$$m_3 = \cos \theta \sin \phi$$

$$n_3 = \cos \theta \cos \phi$$

$$\dot{V}_x = l_1 a_x + m_1 a_y + n_1 a_z$$

$$\dot{V}_y = l_2 a_x + m_2 a_y + n_2 a_z$$

$$\dot{V}_z = l_3 a_x + m_3 a_y + n_3 a_z + g$$

$$u = l_1 V_x + l_2 V_y + l_3 V_z$$

$$v = m_1 V_x + m_2 V_y + m_3 V_z$$

$$w = n_1 V_x + n_2 V_y + n_3 V_z$$

$$V = \left(V_x^2 + V_y^2 + V_z^2\right)^{1/2}$$

$$\alpha = \tan^{-1} \frac{w}{u}$$

$$\beta = \sin^{-1} \frac{v}{V}$$

These additional symbols are also used in the tables and figures.

a_x, a_y, a_z	body axis components of acceleration, m/sec^2
u,v,w	body axis components of velocity, m/sec
V_x, V_y, V_z	inertial axis components of velocity, m/sec

$$M_\alpha = \frac{1}{I_y}\frac{\partial M}{\partial \alpha} \quad \text{per sec}^2\text{-rad}$$

$$M_q = \frac{1}{I_y}\frac{\partial M}{\partial q} \quad \text{per sec-rad}$$

$$M_{\delta_e} = \frac{1}{I_y}\frac{\partial M}{\partial \delta_e} \quad \text{per sec}^2\text{-rad}$$

$$L_\alpha = \frac{1}{mV}\frac{\partial L}{\partial \alpha} \quad \text{per sec-rad}$$

$$L_o = \frac{g}{V_{x_o}} \quad \text{per sec}$$

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TABLE I.- AIRCRAFT OPEN-LOOP CHARACTERISTICS AND STABILITY DERIVATIVES FOR CASES STUDIED IN THIS INVESTIGATION, $L_{\delta_a} = -1.0$

Parameter	Characteristics and derivatives for configuration –					
	AB-1	AB-2	AB-3	BB-1	BB-2	BB-3
ω_d	2.36	2.51	2.55	2.49	2.49	2.6
ζ_d	.12	.10	.11	.10	.10	.08
ϕ/β	1.4	1.6	1.3	5.2	4.8	4.9
T_R	.40	.40	.40	.40	.37	.40
T_S	-91	995	∞	997	987	166
g/V	.055	.055	.055	.055	.055	.055
L'_β	-11.2	-13.3	-10.6	-45.1	-42.1	-40.6
L'_p	-2.67	-2.49	-2.20	-2.68	-2.74	-2.21
L'_r	.901	.73	1.20	1.28	2.06	4.42
N'_β	5.94	6.01	5.29	5.74	5.54	4.85
N'_p	.211	-.013	-.390	.047	.015	-.099
N'_r	-.224	-.351	-.706	-.169	-.278	-.575
Y_β	-.159	-.162	.156	-.156	-.159	-.147
α_o	.021	.024	.025	.022	.021	.024
Case[a]	31, 44, and 62	4, 41, and 45	19, 46, and 56	25, 33, and 57	1, 29, and 38	10, 17, and 47

[a]Each case corresponds to a specific ratio of $N_{\delta_a}/L_{\delta_a}$.

TABLE I.- Continued

Parameter	Characteristics and derivatives for configuration –				
	BC-1	BC-2	BC-3	Group 4	Group 9
ω_d	2.52	2.40	2.36	2.02	1.09
ζ_d	.10	.117	.14	.10	.12
ϕ/β	6.3	7.0	7.5	3.1	3.1
T_R	1.3	1.3	1.6	.40	.40
T_S	-8.3	-16.2	-8.5	100	∞
g/V	.055	.055	.055	.131	.131
L'_β	-41.3	-41.5	-41.6	-19.4	-8.91
L'_p	-.906	-.745	-.176	-2.57	-2.57
L'_r	2.52	4.11	9.33	1.29	.881
N'_β	5.88	5.04	4.66	2.25	.417
N'_p	.078	.044	.002	-.090	-.084
N'_r	-.093	-.377	-.863	-.190	-.042
Y_β	-.171	-.149	-.135	-.151	-.151
α_o	.018	.022	.024	.098	.098
Case[a]	22, 48, and 60	15, 30, and 34	37, 49, and 51	9, 54, and 61	20, 55, and 65

[a]Each case corresponds to a specific ratio of $N_{\delta_a}/L_{\delta_a}$.

TABLE I.- Continued

Parameter	Characteristics and derivatives for configuration –					
	CB-1	CB-2	CB-3	BA-1	BA-2	BA-3
ω_d	2.47	2.48	2.44	2.50	2.54	2.59
ζ_d	.10	.10	.08	.09	.08	.06
ϕ/β	12	14	15	7.4	7.1	7.8
T_R	.40	.46	.40	.15	.15	.125
T_S	-250	111	-104	∞	91	∞
g/V	.055	.055	.055	.055	.055	.055
L'_β	-106	-110	-116	-128	-124	-138
L'_p	-2.86	-2.24	-2.41	-6.71	-6.58	-6.55
L'_r	-.063	7.77	13.9	.854	.920	2.00
N'_β	4.69	4.27	3.06	5.23	4.86	4.26
N'_p	.035	.018	-.011	.009	-.025	-.061
N'_r	.013	-.319	-.345	-.041	-.104	-.062
Y_β	-.142	-.150	-.128	-.152	-.150	-.144
α_o	.022	.020	.022	.021	.022	.022
Case[a]	28, 42, and 64	18, 32, and 58	23, 50, and 59	3, 12, and 39	5, 24, and 43	16, 21, and 40

[a]Each case corresponds to a specific ratio of $N_{\delta_a}/L_{\delta_a}$.

TABLE I.- Continued

Parameter	Characteristics and derivatives for configuration –					
	I-1	I-2	I-11	I-17	I-21	I-30
ω_d	1.93	1.95	2.00	2.00	1.97	(b)
ζ_d	.38	.34	.70	.70	.30	(b)
ϕ/β	1.3	1.3	1.3	1.3	1.3	(b)
$T_R(\omega_{RS})$	.38	(.31)	(.30)	(.40)	(.10)	(b)
$T_S(\zeta_{RS})$	30	(.97)	(.33)	(.91)	(.87)	(b)
g/V	.047	.047	.047	.047	.047	.047
L'_β	-7.14	-7.14	-7.14	-7.14	-7.14	-7.14
L'_p	-2.77	-.554	-.499	-.911	1.20	-5.54
L'_r	.554	.554	2.89	1.38	.026	0
N'_β	3.43	3.43	3.43	3.43	3.43	3.43
N'_p	.053	.053	.218	.158	.531	1.33
N'_r	-1.33	-1.33	-2.45	-2.56	-.133	-2.66
Y_β	-.044	-.044	-.044	-.044	-.044	-.044
α_o	.067	.067	.067	.067	.067	.067
Case[a]	2	26	27	7	35	11

[a]Each case corresponds to a specific ratio of $N_{\delta_a}/L_{\delta_a}$.
[b]All modes are aperiodic.

TABLE I.- Concluded

Parameter	Characteristics and derivatives for configuration –								
	II-1	II-2	II-4	II-14	II-19	II-20	II-21	II-22	II-28
ω_d	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96
ζ_d	.37	.30	.80	.80	.05	.91	.72	.78	.96
$T_R(\omega_{RS})$	.62	(.10)	(.10)	(.40)	(.40)	(.10)	(.39)	(.39)	(.53)
ϕ/β	.80	1.2	1.2	1.0	1.0	2.1	1.3	1.3	1.0
$T_S(\zeta_{RS})$	38	(.30)	(.31)	(.30)	(.70)	(.41)	(.52)	(.89)	(.03)
g/V	.136	.136	.136	.136	.136	.136	.136	.136	.136
L'_β	−1.44	−1.44	−1.44	−1.44	−1.44	−.015	−1.44	−1.44	−1.44
L'_p	−1.65	−.144	−1.08	−.478	−.159	−.974	−.661	−.992	−.144
L'_r	.331	.892	.900	1.05	−.628	−2.20	.734	.734	.891
N'_β	.691	.691	.691	.691	.691	.028	.691	.691	.691
N'_p	0	.148	.592	.230	.024	.084	.233	.233	.148
N'_r	−.555	−.452	−.458	−1.23	−.437	−.752	−1.07	−1.07	−1.59
Y_β	−.027	−.027	−.027	−.027	−.027	−.027	−.027	−.027	−.027
α_o	.262	.262	.262	.262	.262	.262	.262	.262	.262
Case[a]	6	63	52	14	36	66	13	8	53

[a]Each case corresponds to a specific ratio of $N_{\delta_a}/L_{\delta_a}$.

TABLE II.- SYSTEM RESPONSE CHARACTERISTICS TO A BANK-ANGLE COMMAND FOR AIRCRAFT CASES STUDIED, $T_1 = 0.2$ sec

Case	Configuration		Pilot rating from refs. 12, 13, 14	Pilot gain, K_ϕ	Pilot lead, T_2, sec	Closed-loop characteristics of pilot and aircraft							Open-loop aircraft characteristics from table I						
	Group	$N_{\delta_a}/L_{\delta_a}$				ω_1, rad/sec	ζ_1	ω_2, rad/sec	ζ_2	ω_3, rad/sec	ζ_3	Real roots	T_R (ω_{RS}) sec rad/sec	T_S (ζ_{RS})	ω_d, rad/sec	ζ_d	ω_ϕ, rad/sec	ζ_ϕ	ϕ/β
1	BB-2	0	2.0	11	0	2.0	0.09	2.8	0.001	6.9	0.92		0.37	987	2.49	0.10	2.34	0.09	4.8
2	I-1	0	2.0	10	0	1.9	.37	2.4	.003	6.9	.93		.38	30	1.93	.38	1.87	.37	1.3
3	BA-1	0	2.5	32	0	2.1	.07	3.5	.01	9.0	.92		.16	∞	2.50	.09	2.27	.04	7.4
4	AB-2	0	3.0	8	0	2.0	.09	2.6	.06	6.6	.93		.40	995	2.51	.10	2.44	.10	1.6
5	BA-2	0	3.5	32	0	1.9	.13	3.5	0	9.0	.92		.16	91	2.54	.08	2.19	.06	7.1
6	II-1	0	2.5 to 3.5	5.4	0	.83	.38	1.9	0	6.2	.95		.62	38	.98	.37	.88	.35	.8
7	I-17	0	3.5	1.8	0	1.2	0	2.0	.70	5.6	.97		(.40)	(.91)	2.00	.70	1.88	.70	1.8
	I-17	0	3.5	9.5	1.0	1.9	.68	4.9	0			-1.1, -9.9	(.40)	(.91)	2.00	.70	1.88	.70	1.8
8	II-22	0	3.5	3	0	.9	.65	1.5	0	5.7	.96		(.39)	(.89)	1.00	.78	.89	.69	1.3
	II-22	0	3.5	9.8	1.0	.86	.64	5.0	0			-1.0, -10.0	(.39)	(.89)	1.00	.78	.89	.69	1.3
9	Group 4	.05	3.5	8	0	1.56	.18	2.4	0	6.6	.93		.40	100	2.02	.10	1.79	.11	3.14
	Group 4	.05	3.5	12	1.0	1.79	.14	5.5	.07			-.9, -10.7	.40	100	2.02	.10	1.79	.11	3.14
10	BB-3	0	3.0	10	1.0	2.2	.23	5.2	.07			-.8, -10.2	.40	166	2.60	.08	2.20	.16	4.9
11	I-30	0	4.0	26	0	2.0	.67	2.8	0	8.5	.92		(a)	(a)	(a)	(a)	1.88	.74	(a)
12	BA-1	-.03	4.0	24	0	.73	.70	3.6	0	8.6	.92		.16	∞	2.50	.09	.92	.06	7.4
	BA-1	-.03	4.0	32	1.0	.78	.59	4.2	.003	8.1	.97		.16	∞	2.50	.09	.92	.06	7.4
13	II-21	0	4.5	8.5	1.0	.85	.64	4.6	.003			-1.1, -9.4	(.39)	(.52)	1.00	.72	.89	.69	1.2
14	II-14	0	4.5	7.6	1.0	.86	.73	4.4	.008			-1.1, -9.4	(.40)	(.30)	1.00	.80	.89	.78	1.0
15	BC-2	.02	5.0	8	1.0	2.4	.12	4.5	.01			-1.0, -9.5	1.30	-16	2.40	.12	2.39	.13	7.0
16	BA-3	.07	5.5	3	0	2.0	.74	2.3	.01	7.1	.96		.16	∞	2.59	.06	3.78	.06	7.8
17	BB-3	.18	5.5	2	0	1.91	.42	2.3	.04	5.7	.96		.40	166	2.60	.08	3.50	.22	4.9
18	CB-2	0	5.5	8	1.0	2.0	.20	4.9	.12			-.8, -9.8	.46	111	2.48	.10	2.06	.11	14
19	AB-3	0	5.5	12	1.0	2.3	.22	5.5	.04			-.9, -10.6	.40	∞	2.55	.11	2.30	.19	1.3
20	Group 9	.05	5.5	12	1.0	.95	.23	5.5	.07			-.9, -10.7	.40	∞	1.09	.12	.93	.13	3.1
21	BA-3	0	6.0	40	1.0	2.1	.10	8.6	.01			-.9, -15.2	.16	∞	2.59	.06	2.05	.05	7.8
22	BC-1	0	6.0	8	1.0	2.3	.05	4.6	.04			-1.0, -9.5	1.3	-8.3	2.52	.10	2.41	.05	6.3
23	CB-3	0	6.0	16	1.0	1.75	.21	6.3	0			-.9, -11.3	.40	-104	2.44	.08	1.75	.14	14.7
24	BA-2	-.03	6.0	40	1.0	1.05	.50	8.7	.01			-.5, -15.2	.16	91	2.54	.08	.81	.10	7.1

[a]All roots are aperiodic.

TABLE II.- *Continued*

Case	Configuration: Group	Configuration: $N_{\delta_a}/L_{\delta_a}$	Pilot rating from refs. 12, 13, 14	Pilot gain, K_ϕ	Pilot lead, T_2, sec	Closed-loop characteristics of pilot and aircraft: ω_1, rad/sec	ζ_1	ω_2, rad/sec	ζ_2	ω_3, rad/sec	ζ_3	Real roots	Open-loop aircraft characteristics from table I: T_R, sec (ω_{RS}, rad/sec)	T_S (ζ_{RS})	ω_d, rad/sec	ζ_d	ω_ϕ, rad/sec	ζ_ϕ	ϕ/β
25	BB-1	0	6.0	12	0	2.1	0.01	2.8	0.01	7.0	0.92		0.40	997	2.49	0.10	2.38	0.07	5.2
26	I-2	0	4.0 to 6.0	8	1.0	1.9	.37	4.5	0			-1.1, -9.5	(.31)	(.97)	1.95	.34	1.87	.37	1.8
27	I-11	0	6.5	7	1.0	1.9	.64	4.2	.03			-1.0, -9.0	(.30)	(.33)	2.00	.70	1.88	.67	1.4
28	CB-1	0	6.5	12	1.0	2.1	.07	5.7	.09			-.9, -10.8	.40	-250	2.47	.10	2.15	.03	12.0
29	BB-2	-.07	6.5	16	1.0	1.65	.20	6.4	.02			-.8, -11.4	.37	987	2.49	.10	1.65	.08	4.8
30	BC-2	-.07	6.5	8	1.0	1.4	.19	4.9	.02			-1.0, -9.5	1.30	-16.2	2.40	.12	1.51	.07	7.0
31	AB-1	-.07	6.5	12	0	2.1	.0	2.7	.02	7.0	.92		.40	91	2.36	.12	2.27	.07	1.4
32	CB-2	-.03	4.5 to 7.0	14	1.0	.87	.61	6.2	.01			-.5, -10.9	.46	111	2.48	.10	1.75	.135	14.7
33	BB-1	-.10	7.0	16	1.0	.98	.32	3.7	-.04	6.6	.96		.40	997	2.49	.10	1.20	.066	5.2
34	BC-2	.05	7.0	1	1.0	1.8	.75	2.3	.06			-1.4, -6.9	1.30	-16.2	2.40	.12	2.67	.14	7.0
35	I-21	0	7.0	1.8	1.0	1.4	-.03	2.9	.46			-1.3, -7.5	(.10)	(.87)	1.97	.30	1.85	.06	1.8
36	II-19	0	7.0	6.4	1.0	.84	.30	4.0	.01			-1.1, -9.0	(.10)	(.31)	1.0	.80	.88	.39	1.2
37	BC-3	-.08	7.0	6	1.0	1.0	.28	4.5	.05			-1.2, -9.0	1.59	-8.5	2.36	.14	1.18	.10	7.5
38	BB-2	.07	7.0	4	0	2.0	0	2.35	.29	6.2	.95		.37	987	2.49	.10	2.93	.11	4.8
39	BA-1	.07	7.0	1.8	0	2.4	.08	6.8	.97			-1.0, -2.1	.16	∞	2.50	.09	3.55	.04	7.4
40	BA-3	.13	7.0	1.8	0	1.9	.83	2.3	.01	6.9	.97		.16	∞	2.59	.06	4.62	.07	7.8
41	AB-2	.13	7.0	8.0	1.0	2.8	.05	4.6	.21			-.9, -9.9	.40	995	2.51	.10	2.77	.11	1.6
42	CB-1	.015	7.0	16	1.0	2.4	.02	6.2	.04			-.9, -11.5	.40	-250	2.47	.10	2.49	.03	12.0
43	BA-2	.07	7.0	1.2	1.0	2.5	0	4.8	.82			-.3, -8.6	.16	91	2.54	.08	3.72	.06	7.1
44	AB-1	-.17	7.0	18	1.0	2.0	.09	6.5	0			-.9, -11.7	.40	-91	2.36	.12	2.00	.05	1.4
45	AB-2	-.16	7.0	16	1.0	2.0	.16	6.3	0			-.9, -11.4	.40	995	2.51	.10	1.97	.09	1.6
46	AB-3	-.13	7.0	20	1.0	2.0	.24	6.0	.01			-.9, -11.0	.40	∞	2.55	.11	1.98	.18	1.3

TABLE II.- Concluded

Case	Configuration		Pilot rating from refs. 12, 13, 14	Pilot gain, K_ϕ	Pilot lead, T_2, sec	Closed-loop characteristics of pilot and aircraft							Open-loop aircraft characteristics from table I						
	Group	$N_{\delta_a}/L_{\delta_a}$				ω_1, rad/sec	ζ_1	ω_2, rad/sec	ζ_2	ω_3, rad/sec	ζ_3	Real roots	T_R, (ω_{RS}), sec rad/sec	T_S (ζ_{RS})	ω_d, rad/sec	ζ_d	ω_ϕ, rad/sec	ζ_ϕ	ϕ/β
47	BB-3	-0.111	7.5	12	1.0	1.08	0.69	6.0	0.04			-0.24, -10.7	0.4	166	2.6	0.08	0.59	0.15	4.9
48	BC-1	-.03	7.5	8	1.0	2.0	.08	4.8	.03			-1.0, -9.5	1.3	-8.3	2.52	.10	2.10	.04	6.3
49	BC-3	0	7.5	6	1.0	2.1	.28	4.1	0			-1.1, -8.9	1.6	-8.5	2.36	.14	2.17	.23	7.5
50	CB-3	-.03	7.5			No stable solution													
51	BC-3	.07	8.0	.8	1.0	1.3	.75	2.3	.02			-2.6, -6.5	1.6	-8.5	2.36	.14	2.80	.31	7.5
52	II-4	0	8.0	10	1.0	.83	.27	5.0	.01			-1.0, -10.0	(.10)	(.31)	1.00	.80	.88	.39	1.2
53	II-28	0	8.0	6.4	1.0	.87	.92	4.0	0			-1.1, -9.0	(.53)	(.03)	.96	.96	.90	.98	1.0
54	Group 4	-.05	8.0	12	1.0	1.17	.29	5.7	.06			-.8, -10.7	.4	100	2.02	.10	1.13	.12	3.4
55	Group 9	0	8.0	12	1.0	.69	.35	5.5	.07			-.8, -10.7	.4	∞	1.09	.12	.64	.15	3.1
56	AB-3	.18	8.0	5	1.0	2.1	.38	2.5	0	5.9	.97		.4	∞	2.55	.11	2.68	.21	1.3
57	BB-1	.05	8.0	2	1.0	2.5	.02	3.6	.61			-.6, -7.9	.4	997	2.49	.10	2.82	.07	5.2
58	CB-2	.04	8.0	2	1.0	2.5	.01	3.4	.61			-.7, -7.8	.46	111	2.48	.10	2.88	.14	14
59	CB-3	.06	8.0	.8	1.0	2.5	0	3.3	.83			-.4, -6.9	.4	-104	2.44	.08	3.14	.22	15
60	BC-1	.04	8.0	2	1.0	2.4	.01	2.8	.45			-1.1, -7.6	1.3	-8.3	2.52	.10	2.70	.07	6.3
61	Group 4	0	8.0	12	1.0	1.50	.19	5.6	.06			-.9, -10.7	.4	100	2.02	.10	1.50	.11	3.1
62	AB-1	0	8.0	18	1.0	2.4	.07	6.4	0			-.9, -11.7	.4	-91	2.36	.12	2.42	.08	1.4
63	II-2	0	7.0 to 8.0	6.5	1.0	.83	.30	4.0	0			-1.1, -9.0	(.10)	(.30)	1.00	.30	.88	.31	1.2
64	CB-1	-.07	9.0			No stable solution													
65	Group 9	-.05	9.0			No stable solution													
66	II-20	0	10.0			No stable solution													

TABLE III.- SYSTEM RESPONSE CHARACTERISTICS TO A HEADING COMMAND

Case	Configuration		K_ϕ	T_1, sec	T_2, sec	$K_\psi \frac{g}{V}$	Closed-loop characteristics							Computed pilot rating	Experimental pilot rating
	Group	$N_{\delta_a}/L_{\delta_a}$					ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots		
1	BB-2	0	10	0.2	0	0.6	1.9	0	2.6	0	6.7	0.93	-0.68	1 to 3.5	2
2	I-1	0	8	.2	0	.5	1.87	.4	2.0	-.04	6.6	.94	-.56	1 to 3.5	2
3	BA-1	0	28	.2	0	.6	1.7	0	2.0	.03	8.7	.92	-.68	1 to 3.5	2.5
4	AB-2	0	6	.2	0	.6	1.7	0	2.5	.07	6.3	.94	-.72	1 to 3.5	3
5	BA-2	0	22	.2	0	.6	1.7	.12	3.2	0	8.4	.93	-.73	1 to 3.5	3.5
14	II-14	0	6.5	.2	1	.37	.9	.73	4.0	0			-.4, -1.1, -9.0	3.5 to 6.5	4.5
19	AB-3	0	3	.2	1	2.0	2.1	.26	4.0	0			-1.4, -1.7, -8.9	3.5 to 6.5	5.5
26	I-2	0	6.5	.2	1	.4	1.9	.36	4.0	0			-.4, -1.0, -9.0	3.5 to 6.5	6
25	BB-1	0	8	.2	1	1.8	2.3	.07	4.5	.03			-1.3, -1.5, -9.6	3.5 to 6.5	6
31	AB-1	-.07	8	.2	1	2.0	2.2	.06	4.5	.02			-1.1, -1.8, -9.5	3.5 to 6.5	6.5
46	AB-3	-.13	6	.05	1	26	1.7	.02	4.0	.01	20	.99	-1.2	6.5 to 9	7
38	BB-2	.07	1.25	.05	1	20	3.3	.08	4.2	.04			-1.0, -20, -21	6.5 to 9	7

TABLE IV.- HEADING RESPONSE CHARACTERISTICS FOR UNUSUAL CASES

Case	Configuration		K_ϕ	T_1, sec	T_2, sec	$K_\psi \frac{g}{V}$	Closed-loop characteristics						
	Group	$N'_{\delta_a}/L'_{\delta_a}$					ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots
25	BB-1	0	4	0.2	0	0.8	1.3	-0.01	2.5	0.09	6.0	0.95	-1.1
			8	.2	1	1.8	2.3	.07	4.5	.03			-1.3,-1.5,-9.6
19	AB-3	0	3	0.2	0	1.2	1.6	0.04	2.5	0	6.1	0.94	-1.4
			3	.2	1	2.0	2.1	.26	4.0	0			-1.4,-1.7,-8.9
38	BB-2	0.07	3	0.2	0	0.6	1.5	0.04	2.4	0.17	6.0	0.96	-0.82
			2	.2	1	.6	.7	.75	2.5	-.01	3.4	.64	-7.8
			1.25	.05	1	20	3.3	.08	4.2	.04			-1,-20,-21
39	BA-1	0.06	4.3	0.2	0	0.6	1.8	0.41	2.0	0.09	7.2	0.96	-1.1
			1.5	.2	1	.6	.5	.52	2.4	0	4.7	.81	-8.8
			.5	.05	1	4	.8	.41	2.4	.01			-1.6,-16,-23
			3	.05	0	1.5	1.6	.38	2.2	0	20	.99	-5.5
17	BB-3	0.18	2	0.2	0	0.8	1.6	0	2.3	0.17	5.6	0.91	-1.1
			.6	.2	1	1.8	.9	.49	2.4	0	2.9	.95	-6.4
			.3	.05	1	2.5	.75	.37	2.5	.02			-2.5,-17,-22
			1.25	.05	0	2.2	1.6	0	2.2	.09	20	.99	-2.4

TABLE IV.- Concluded

Case	Configuration		K_ϕ	T_1, sec	T_2, sec	$K_\psi \frac{g}{V}$	Closed-loop characteristics						
	Group	$N'_{\delta_a}/L'_{\delta_a}$					ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots
16	BA-3	0.07	3	0.2	0	1.2	1.4	0.04	2.2	0.13	6.9	0.97	-2.4
			.5	.2	1	6.0	1.2	.36	2.3	0			-3.1,-5.8,-6.9
			2	.05	1	10	.85	.37	2.5	0			-6,-10,-21
			2.5	.05	0	2	1.5	.19	2.2	0	20	.99	-5.8
43	BA-2	0.07	2	0.2	0	1.4	1.1	0.01	2.3	0.13	6.8	0.98	-3
			1	.2	1	1.4	.7	.44	2.4	0	4.5	.87	-8.3
			.3	.05	1	8	.95	.43	2.4	0			-6,-17,-22
			2.5	.05	0	2	1.6	.19	2.1	.02	20	.99	-5.7

TABLE V.- ROLL RESPONSE CHARACTERISTICS FOR UNUSUAL CASES

Case	Configuration		K_ϕ	T_2, sec	Closed-loop characteristics						
	Group	$N'_{\delta_a}/L'_{\delta_a}$			ω_1	ζ_1	ω_2	ζ_2	ω_3	ζ_3	Real roots
16	BA-3	0.07	3	0	2.0	0.74	2.3	0.01	7.1	0.96	
			17	1	2.5	0	4.7	.87			-0.23, -8.6
38	BB-2	0.07	2	1	2.5	0	3.7	0.63			-0.6, -7.9
			4	1	2.7	-.03	4.1	.46			-.8, -8.8
			10	1	3.0	.02	5.0	.17			-.9, -10.4

TABLE VI.- COMPUTED PILOT RATINGS

Case	Configuration		Experimental pilot rating	Computed pilot rating	Comments
	Group	$N_{\delta_a}/L_{\delta_a}$			
[a]1	BB-2	0	2.0	1 to 3.5	Simple rule applies
[a]2	I-1	0	2.0	1 to 3.5	
[a]3	BA-1	0	2.5	1 to 3.5	
[a]4	AB-2	0	3.0	1 to 3.5	
[a]5	BA-2	0	3.5	1 to 3.5	
6	II-1	0	2.5 to 3.5	1 to 3.5	Pole-zero cancellation
7	I-17	0	3.5	3.5	Pole-zero cancellation, borderline cases
8	II-22	0	3.5	3.5	
9	Group 4	.05	3.5	3.5	
10	BB-3	0	3.0	3.5 to 6.5	Incorrect prediction
[a]26	I-2	0	4.0 to 6.0	3.5 to 6.5	Simple rule applies
15	BC-2	.02	5.0	3.5 to 6.5	
18	CB-2	0	5.5	3.5 to 6.5	
[a]19	AB-3	0	5.5	3.5 to 6.5	
21	BA-3	0	6.0	3.5 to 6.5	
22	BC-1	0	6.0	3.5 to 6.5	
27	I-11	0	6.5	3.5 to 6.5	
28	CB-1	0	6.5	3.5 to 6.5	

[a]Cases for which heading as well as roll-control pilot ratings were made.

TABLE VI.- Continued

Case	Configuration		Experimental pilot rating	Computed pilot rating	Comments
	Group	$N_{\delta_a}/L_{\delta_a}$			
13	II-21	0	4.5	3.5 to 6.5	Pole-zero cancellation
[a]14	II-14	0	4.5	3.5 to 6.5	
20	Group 9	.05	5.5	3.5 to 6.5	
23	CB-3	0	6.0	3.5 to 6.5	
29	BB-2	-.07	6.5	3.5 to 6.5	
30	BC-2	-.07	6.5	3.5 to 6.5	
[a]25	BB-1	0	6.0	3.5 to 6.5	Heading control the critical factor
[a]31	AB-1	-.07	6.5	3.5 to 6.5	
11	I-30	0	4.0	1 to 3.5	Incorrect prediction, unusual system response
12	BA-1	-.03	4.0	6.5 to 9.0	
32	CB-2	-.03	4.5 to 7.0	6.5 to 9.0	
24	BA-2	-.03	6.0	6.5 to 9.0	
[a]16	BA-3	0.07	5.5	6.5 to 9.0	Incorrect prediction, heading control the critical factor
[a]17	BB-3	.18	5.5	6.5 to 9.0	

[a]Cases for which heading as well as roll-control pilot ratings were made.

TABLE VI.- Continued

Case	Configuration		Experimental pilot rating	Computed pilot rating	Comments
	Group	$N_{\delta_a}/L_{\delta_a}$			
33	BB-1	-0.10	7.0	6.5 to 9.0	Simple rule applies
34	BC-2	.05	7.0	6.5 to 9.0	
35	I-21	0	7.0	6.5 to 9.0	
36	II-19	0	7.0	6.5 to 9.0	
37	BC-3	-.08	7.0	6.5 to 9.0	
47	BB-3	-.111	7.5	6.5 to 9.0	
63	II-2	0	7.0 to 8.0	6.5 to 9.0	
51	BC-3	.07	8.0	6.5 to 9.0	
52	II-4	0	8.0	6.5 to 9.0	
53	II-28	0	8.0	6.5 to 9.0	
54	Group 4	-.05	8.0	6.5 to 9.0	
55	Group 9	0	8.0	6.5 to 9.0	
[a]38	BB-2	0.07	7.0	6.5 to 9.0	Heading control the critical factor
[a]39	BA-1	.07	7.0	6.5 to 9.0	
40	BA-3	.13	7.0	6.5 to 9.0	
41	AB-2	.13	7.0	6.5 to 9.0	
42	CB-1	.015	7.0	6.5 to 9.0	
[a]43	BA-2	.07	7.0	6.5 to 9.0	
[a]46	AB-3	-.13	7.0	6.5 to 9.0	
56	AB-3	.18	8.0	6.5 to 9.0	
57	BB-1	.05	8.0	6.5 to 9.0	
58	CB-2	.04	8.0	6.5 to 9.0	
59	CB-3	.06	8.0	6.5 to 9.0	
60	BC-1	.04	8.0	6.5 to 9.0	

[a]Cases for which heading as well as roll-control pilot ratings were made.

TABLE VI.- Concluded

Case	Configuration		Experimental pilot rating	Computed pilot rating	Comments
	Group	$N_{\delta_a}/L_{\delta_a}$			
61	Group 4	0	8.0	3.5 to 6.5	Incorrect prediction, pole-zero cancellation
44	AB-1	-0.17	7.0	3.5 to 6.5	Incorrect prediction
62	AB-1	0	8.0	3.5 to 6.5	
45	AB-2	-.16	7.0	3.5 to 6.5	
48	BC-1	-.03	7.5	3.5 to 6.5	
49	BC-3	0	7.5	3.5 to 6.5	
50	CB-3	-0.03	7.5	9.0 to 10.0	Simple rule applies
64	CB-1	-.07	9.0	9.5 to 10.0	
65	Group 9	-.05	9.0	9.0 to 10.0	
66	II-20	0	10.0	9.0 to 10.0	

TABLE VII.- MEASURED PILOT RESPONSE

Case	Configuration		Pilot B	Pilot M	Pilot K
	Group	$N_{\delta_a}/L_{\delta_a}$			
1	BB-2	0	$\frac{\delta_a}{\phi_e} = \frac{3.33(1 + 0.22s)}{(1 + 0.22s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{2.56(1 + 0.09s)}{(1 + 0.21s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{1.2}{(1 + 0.2s)^2}$
38	BB-2	0.07	$\frac{\delta_a}{\phi_e} = \frac{1.88(1 + 0.5s)}{(1 + 0.12s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{1.5(1 + 0.25s)}{(1 + 0.17s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{0.9(1 + 0.04s)}{(1 + 0.2s)^2}$
33	BB-1	-0.10	$\frac{\delta_a}{\phi_e} = \frac{5.0(1 + 0.38s)}{(1 + 0.2s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{1.8(1 + 0.2s)}{(1 + 0.18s)^2}$	$\frac{\delta_a}{\phi_e} = \frac{3.5}{(1 + 0.22s)^2}$

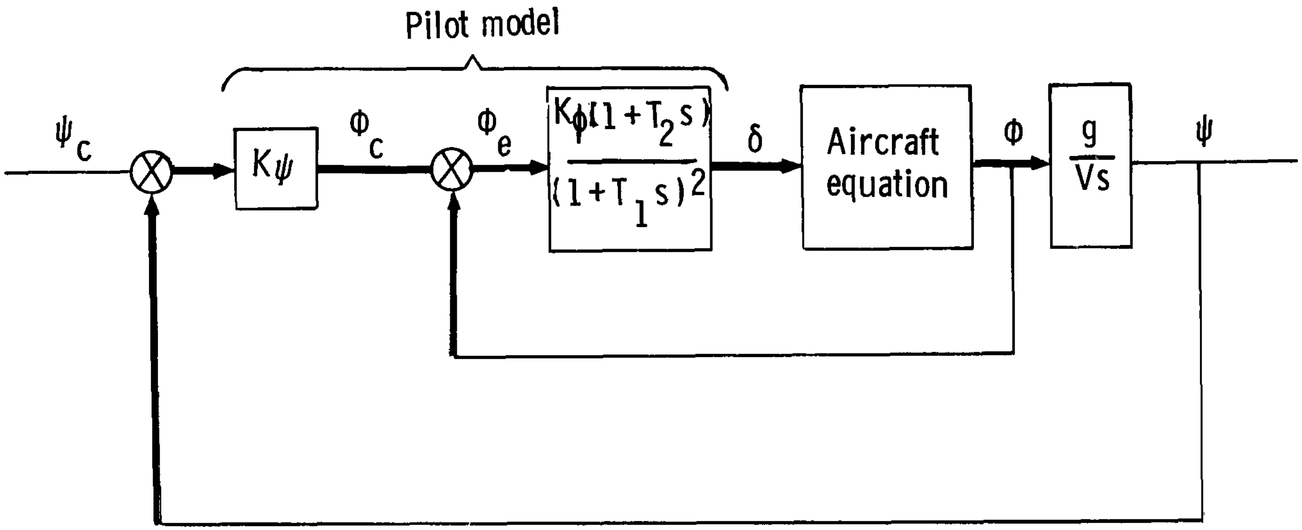


Figure 1.- Block diagram of heading control system.

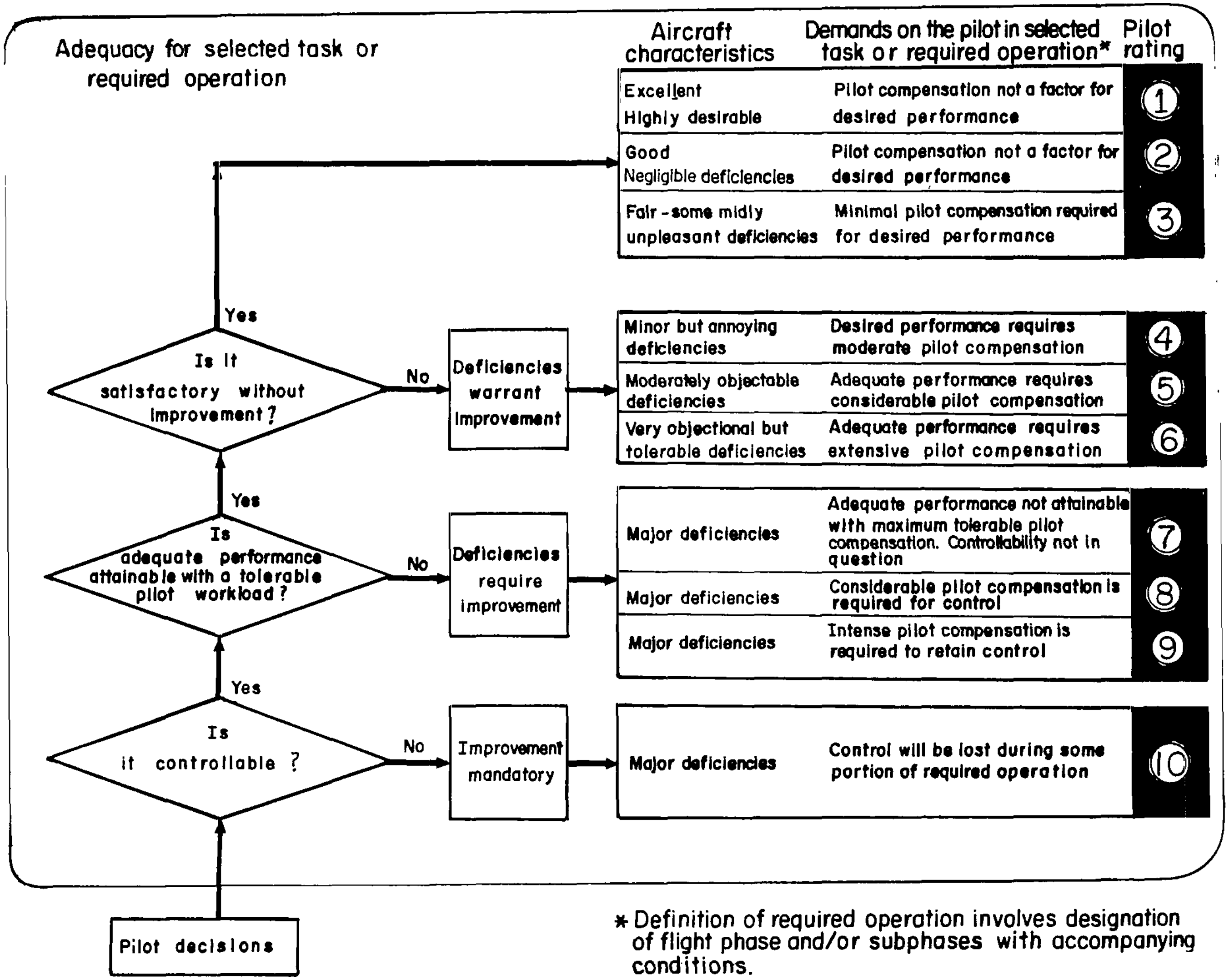


Figure 2.- Pilot rating chart.

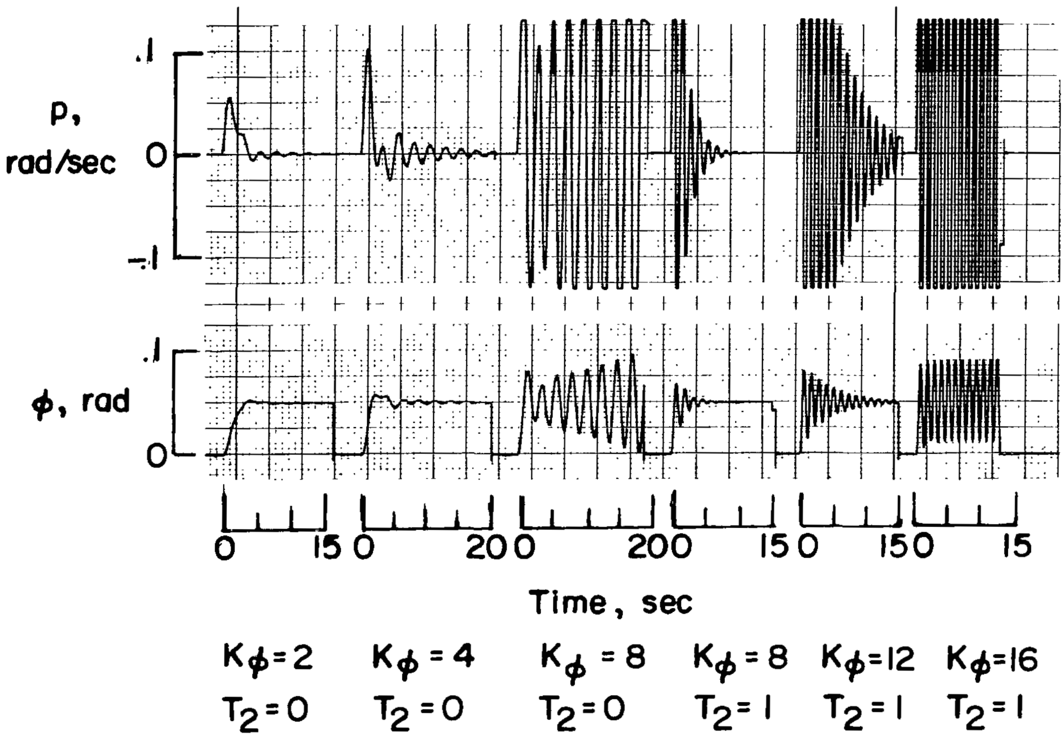
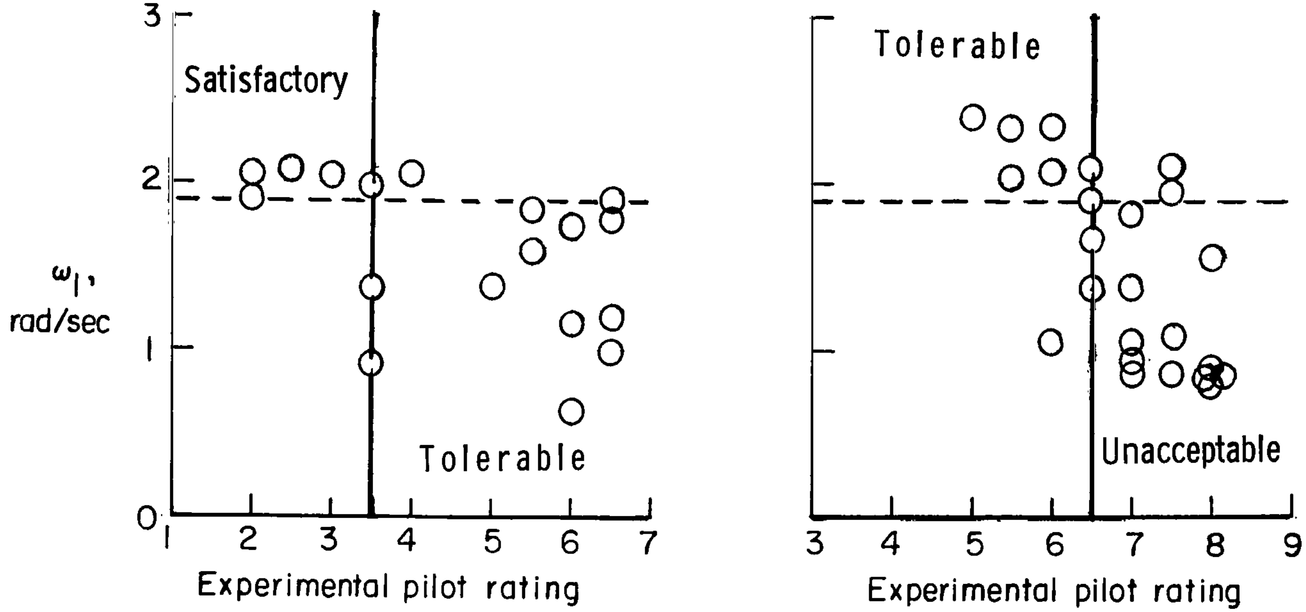


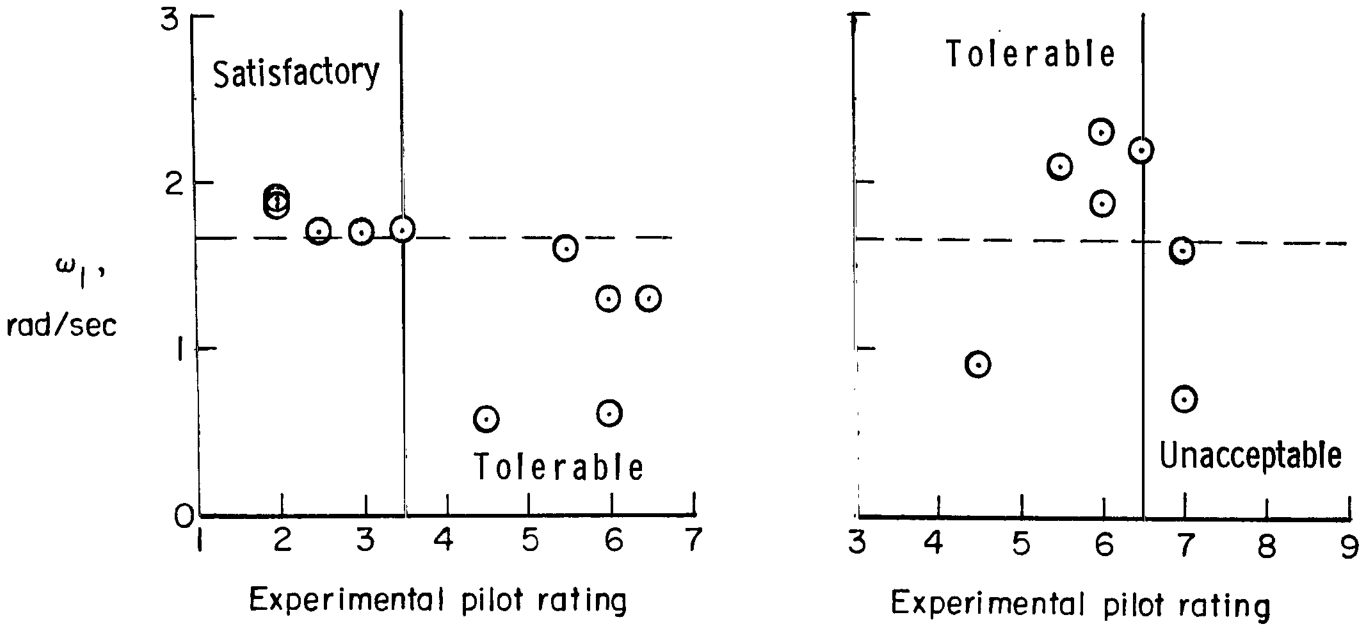
Figure 3.- Time histories of roll-control system response to step command with various pilot model gains. Aircraft configuration is AB-3. $\frac{N_{\delta_a}}{L_{\delta_a}} = 0$; case 19.



(a) No pilot-model lead.

(b) With pilot-model lead.

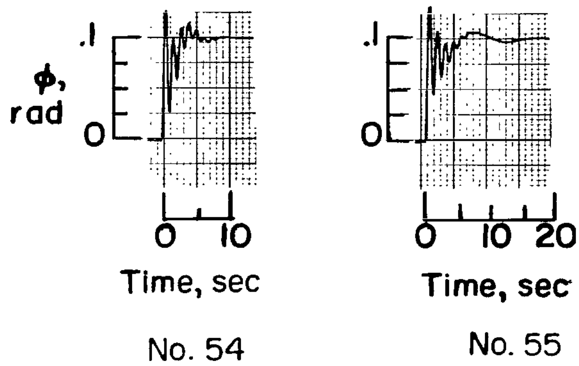
Figure 4.- Lowest system frequency plotted against experimentally obtained pilot ratings for roll control.



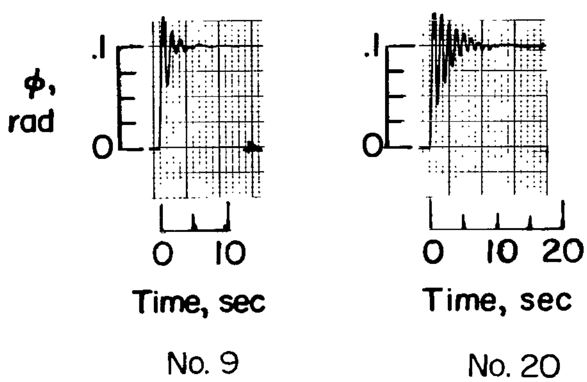
(a) No pilot-model lead.

(b) With pilot-model lead.

Figure 5.- Lowest system frequency plotted against experimentally obtained pilot ratings for heading control.

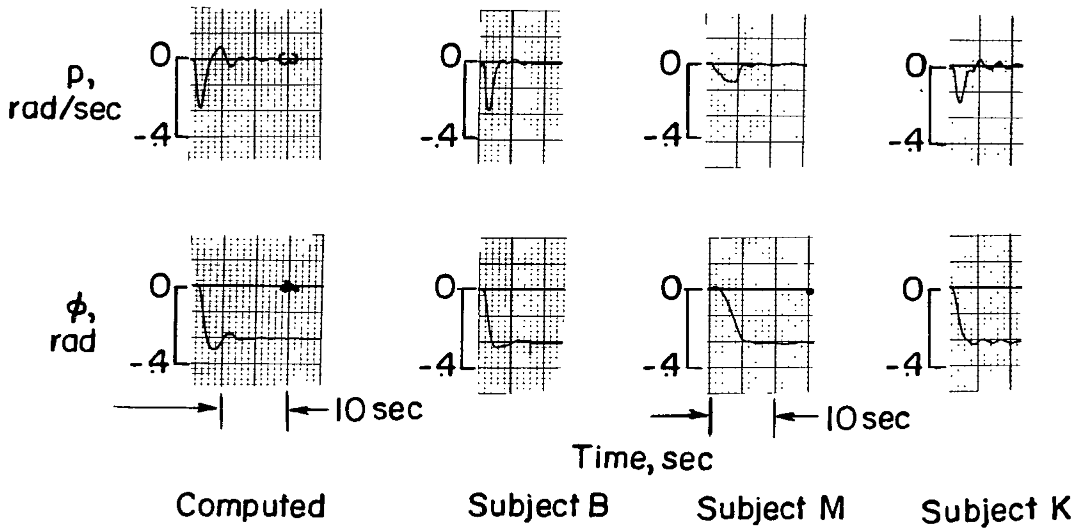


(a) Configurations for which pole-zero cancellation does not occur.



(b) Configurations for which pole-zero cancellation does occur.

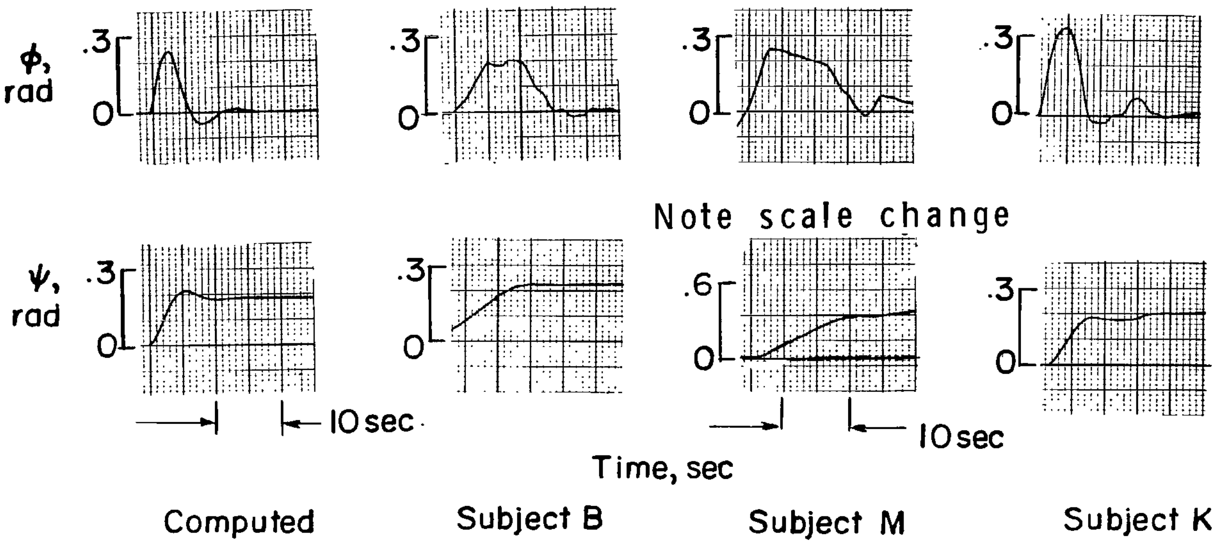
Figure 6.- Time histories of roll-control system response illustrating pole-zero cancellation.



$$\frac{\delta_a}{\phi} = \frac{4}{(1 + 0.2s)^2}$$

(a) Response to a step ϕ command.

Figure 7.- Computed and piloted system responses with configuration BB-2. $\frac{N_{\delta_a}}{L_{\delta_a}} = 0$; case 1.

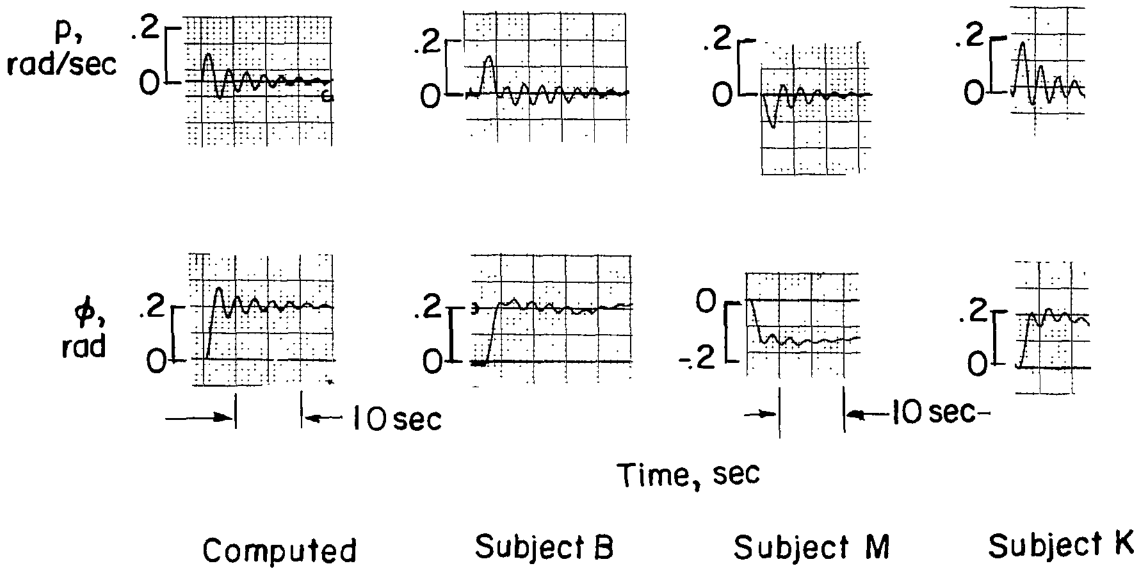


$$\frac{\delta_a}{\phi_e} = \frac{2}{(1 + 0.2s)^2}$$

$$\frac{\phi_c}{\psi_e} = 0.4\,\frac{V}{g}$$

(b) Response to a step ψ command.

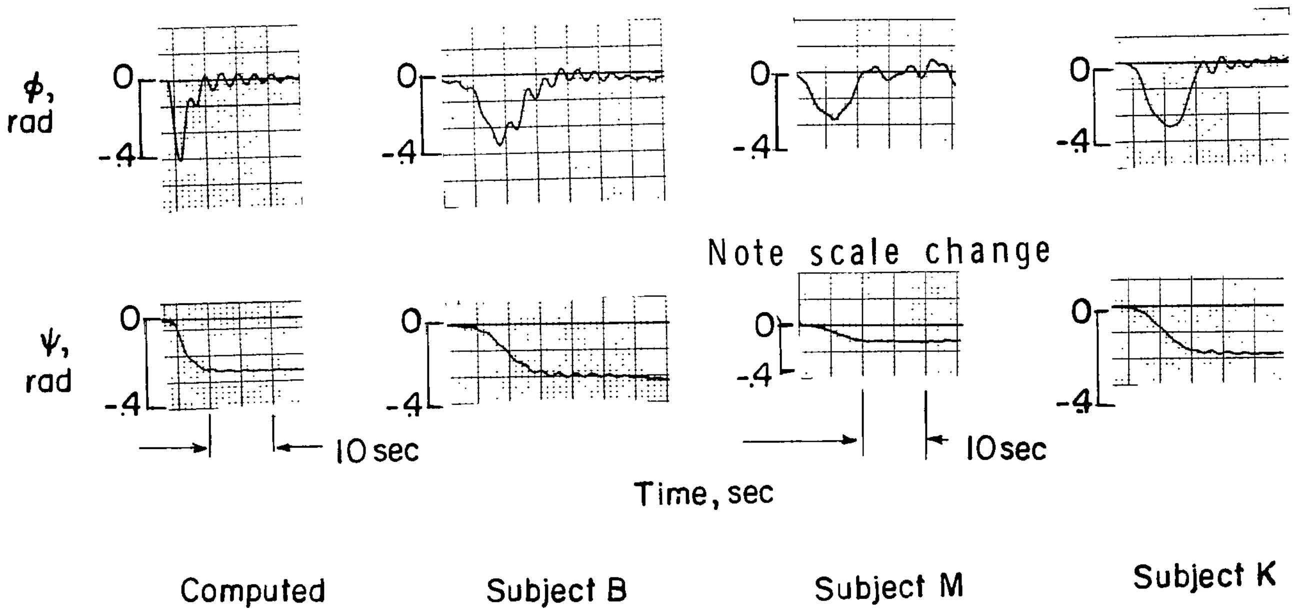
Figure 7.- Concluded.



$$\frac{\delta_a}{\phi_e} = \frac{3(1 + 0.2s)}{(1 + 0.2s)^2}$$

(a) Response to a step ϕ command.

Figure 8.- Computed and piloted system responses with configuration BB-2. $\frac{N_{\delta_a}}{L_{\delta_a}} = 0.07$; case 38.

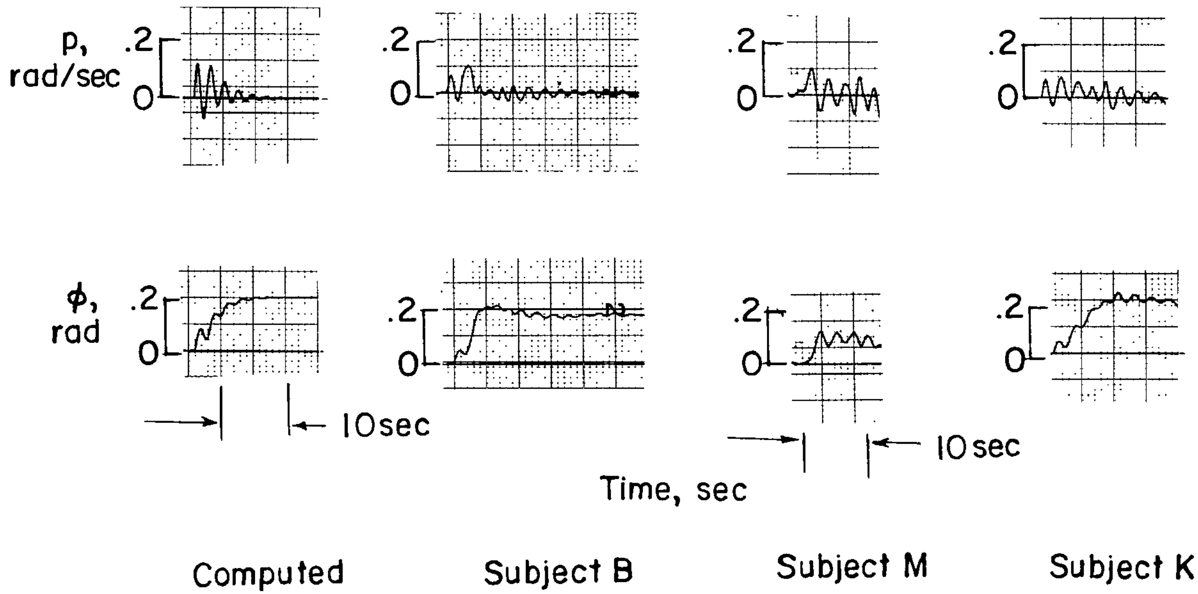


$$\frac{\delta_a}{\phi_e} = \frac{2.5(1 + 0.4s)}{(1 + 0.2s)^2}$$

$$\frac{\phi_c}{\psi_e} = 0.4 \frac{V}{g}$$

(b) Response to a step ψ command.

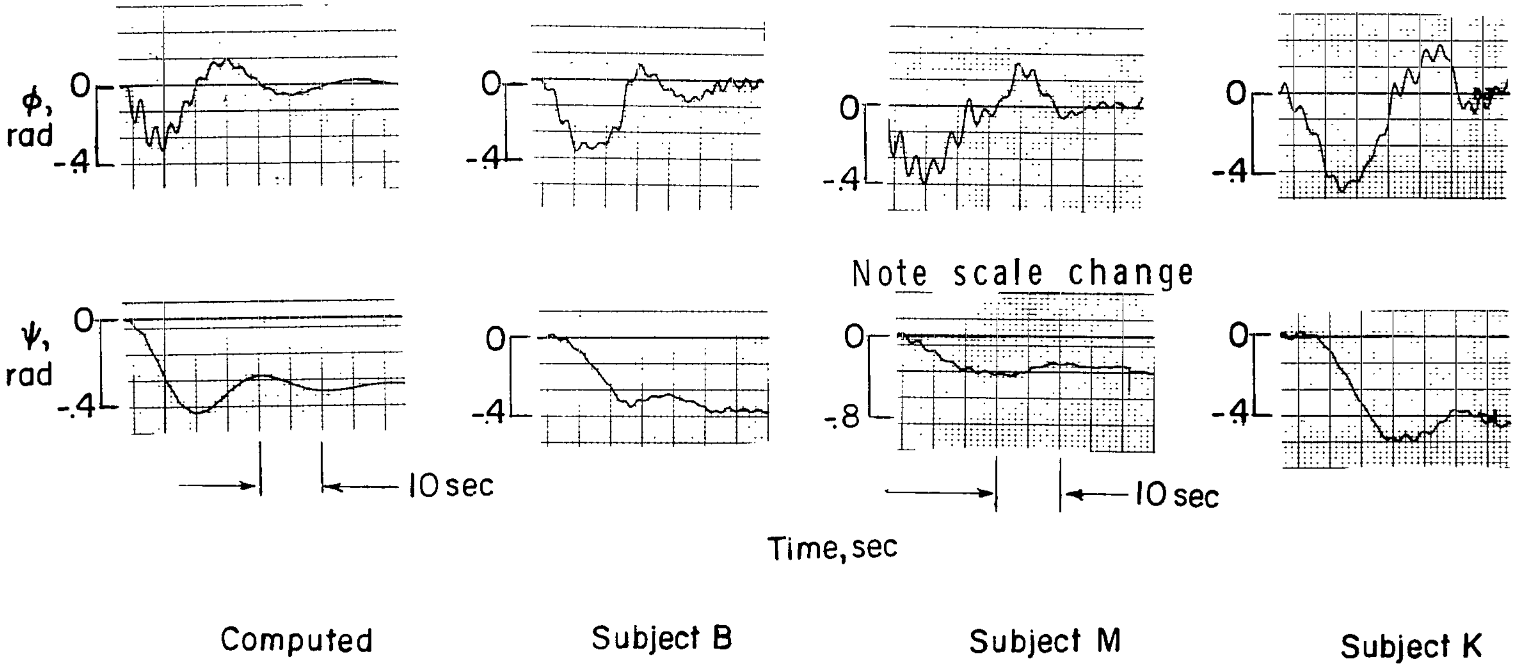
Figure 8.- Concluded.



$$\frac{\delta_a}{\phi_e} = \frac{4(1 + 0.2s)}{(1 + 0.2s)^2}$$

(a) Response to a step ϕ command.

Figure 9.- Computed and piloted system responses with configuration BB-1. $\frac{N_{\delta_a}}{L_{\delta_a}} = -0.10$; case 33.



$$\frac{\delta_a}{\phi_e} = \frac{3}{(1 + 0.2s)^2}$$

$$\frac{\phi_c}{\psi_e} = 0.4\,\frac{V}{g}$$

(b) Response to a step ψ command.

Figure 9.- Concluded.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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